Math 1231: Single-Variable Calculus I
George Washington University Fall 2026
Recitation 5

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We can use the limit definition of the derivative to compute derivatives, but it's not very efficient. In class we learned some rules for computing them more easily:

- (Constants) $\frac{d}{dx}c = 0$
- (Identity) $\frac{d}{dx}x = 1$
- (Scalar products) $\frac{d}{dx}cf(x) = cf'(x)$
- (Sum Rule) $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$
- (Power Rule) $\frac{d}{dx}x^n = nx^{n-1}$
- (Product Rule) $\frac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$
- (Quotient Rule) $\dfrac{d}{dx}\dfrac{f(x)}{g(x)} = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$.

Problem 1. Let's find the derivative of $3x^2 + 5x$.

(a) First just plug 1 into this function. Think about what operations you do in which order. Which operation do you do last?

(b) What rule should we invoke first? Use that rule to write this as a combination of two smaller derivative problems.

(c) What rule do we use next? We need to use it in two places.

(d) Finish off computing this derivative. What rules did you use?

Solution:

(a) When we plug in 1 we get $3(1)^2 + 5(1) = 3 + 5 = 8$. The last thing we did was add two numbers.

(b) We need to use the sum rule first. We get

$$\frac{d}{dx}3x^2 + 5 = (3x^2)' + (5x)'.$$

(c) The scalar product rule gives us

$$(3x^2)' + (5x)' = 3(x^2)' + 5(x)'$$

(d)

$$3(x^2)' + 5(x)' = 3(2x) + 5.$$

This uses the power rule and the identity rule. (Technically we could just say the power rule if we really wanted.)

Problem 2. (a) Use the product rule to differentiate $(x^2 + 1)(3x^3 - 5)$.

(b) Multiply out $(x^2 + 1)(3x^3 - 5)$ to get one big polynomial. Use our derivative rules to compute that derivative.

(c) Which process was easier?

Solution:

(a) $2x(3x^3 - 5) + (x^2 + 1)(9x^2)$.

(b) We get

$$\begin{aligned}(x^2 + 1)(3x^3 - 5) &= 3x^5 - 5x^2 + 3x^3 - 5\\ \frac{d}{dx}(x^2 + 1)(3x^3 - 5) &= \frac{d}{dx}3x^5 - 5x^2 + 3x^3 - 5\\ &= 15x^4 - 10x + 9x^2 - 0.\end{aligned}$$

(c) This is a matter of personal taste, but I'd say the second derivative was easier, but took more work total when we count the work of multiplying the terms out.

Problem 3. Compute $\dfrac{d}{dx}\dfrac{x^5 - 7x}{4x^2 + 3}$.

Solution:

$$\frac{d}{dx}\frac{x^5-7x}{4x^2+3} = \frac{(5x^5-7)(4x^2+3)-(8x)(x^5-7x)}{(4x^2+3)^2}.$$

Problem 4. Consider the function $\sec^2(x^2+1)$

(a) Find functions f and g so that $(f\circ g)(x) = \sec^2(x^2+1)$.

(b) Talk to the people next to you. Did they pick the same f and g that you did? Can you find a different pair of functions f and g that also work?

(c) Find functions f, g, h so that $(f\circ g\circ h)(x) = \sec^2(x^2+1)$.

(d) Compute f', g', and h'.

(e) What is $\frac{d}{dx}\sec^2(x^2+1)$?

Solution:

(a) There are basically two choices here. You could say that $f(x) = \sec^2(x)$ and $g(x) = x^2+1$, which is maybe the more obvious choice; or you could say that $f(x) = x^2$ and $g(x) = \sec(x^2+1)$.

(b) This is really a composite of three functions, which is why you could make different choices here.

(c) $f(x) = x^2, g(x) = \sec(x), h(x) = x^2+1$. (Technically there are other things you could do, like $g(x) = \sec(x+1)$ and $h(x) = x^2$, but those are moderately silly.)

(d) $f'(x) = 2x, g'(x) = \sec(x)\tan(x), h'(x) = 2x$.

(e)

$$\begin{aligned}\frac{d}{dx}\sec^2\left(x^2+1\right) &= f'(g(h(x))\cdot g'(h(x))\cdot h'(x)\\ &= f'(\sec(x^2+1))\cdot g'(x^2+1)\cdot h'(x)\\ &= 2\sec(x^2+1)\cdot\sec(x^2+1)\tan(x^2+1)\cdot 2x.\end{aligned}$$

Problem 5. Find

$$\frac{d}{dx}\frac{\sin(x^2)+\sin^2(x)}{x^2+1}$$

Solution:

$$\begin{aligned}\frac{d}{dx}\frac{\sin(x^2)+\sin^2(x)}{x^2+1} &= \frac{(\sin(x^2)+\sin^2(x))'(x^2+1)-2x(\sin(x^2)+\sin^2(x))}{(x^2+1)^2}\\ &= \frac{(\cos(x^2)\cdot 2x+2\sin(x)\cos(x))(x^2+1)-2x(\sin(x^2)+\sin^2(x))}{(x^2+1)^2}.\end{aligned}$$

Problem 6. (a) Compute

$$\frac{d}{dx}\sqrt{\frac{\sqrt{x}+1}{(\cos x+1)^2}}$$

Solution:

$$\begin{aligned}\frac{d}{dx}\sqrt{\frac{\sqrt{x}+1}{(\cos x+1)^2}} &= \frac{1}{2}\left(\frac{\sqrt{x}+1}{(\cos x+1)^2}\right)^{-1/2}\cdot\left(\frac{\sqrt{x}+1}{(\cos x+1)^2}\right)'\\ &= \frac{1}{2}\left(\frac{\sqrt{x}+1}{(\cos x+1)^2}\right)^{-1/2}\cdot\frac{\frac{1}{2}x^{-1/2}(\cos x+1)^2-2(\cos x+1)(-\sin x)(\sqrt{x}+1)}{(\cos x+1)^4}\end{aligned}$$

(b) Find

$$\frac{d}{dx}\tan^4(\sqrt[3]{x^5+x^3+2}+1).$$

Solution:

$$\begin{aligned}\frac{d}{dx}\tan^4(\sqrt[3]{x^5+x^3+2}+1) &= 4\tan^3(\sqrt[3]{x^5+x^3+2}+1)\cdot\sec(\sqrt[3]{x^5+x^3+2}+1)\\ &\quad\cdot\tan(\sqrt[3]{x^5+x^3+2}+1)\cdot(\sqrt[3]{x^5+x^3+2}+1)'\\ &= 4\tan^4(\sqrt[3]{x^5+x^3+2}+1)\sec(\sqrt[3]{x^5+x^3+2}+1)\\ &\quad\cdot\left(\frac{1}{3}(x^5+x^3+1)^{-2/3}\cdot(5x^4+3x^2)\right).\end{aligned}$$

Problem 7 (Bonus)**.** Calculate

$$\frac{d}{dx}\left(\frac{\sin^2\left(\frac{x^2+1}{\sqrt{x-1}}\right)+\sqrt{x^3-2}}{\cos(\sqrt{x^2+1}+1)-\tan(x^4+3)}\right)^{5/3}$$