Math 1231 Fall 2026
Single-Variable Calculus I Section 11
Mastery Quiz Worksheet 5
Due Friday, September 25

This week's mastery quiz has three topics. It is the first time we have seen topic M2, and the last quiz featuring M1 or S2.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, you should justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

On Friday, we will end class with a short quiz, containing questions drawn from this worksheet, plus one "easy" question on topic M1. You may not use or consult any materials or notes during Friday's quiz; make sure you work through these problems carefully, and understand them well enough that you can replicate that work in class.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Major Topic 2: Computing Derivatives
- Secondary Topic 2: Definition of Derivative

Major Topic 1: Computing Limits

(a) Compute $\lim_{x\to 0} \dfrac{x\sin(2x)}{\sin(5x)\sin(3x)} =$

Solution:

$$\lim_{x\to 0} \frac{x\sin(2x)}{\sin(5x)\sin(3x)} = \lim_{x\to 0} \frac{5x}{\sin(5x)} \frac{\sin(2x)}{2x} \frac{3x}{\sin(3x)} \frac{2}{15} = \frac{2}{15}.$$

(b) Compute $\lim_{x\to 3} \dfrac{1}{x-3} - \dfrac{3}{x^2-3x} =$

Solution:

$$\begin{aligned}\lim_{x\to 3} \frac{1}{x-3} + \frac{3}{x^2-3x} &= \lim_{x\to 3} \frac{x^2-3x-3(x-3)}{(x-3)(x^2-3x)} \\ &= \lim_{x\to 3} \frac{x^2-6x+9}{x(x-3)^2} \\ &= \lim_{x\to 3} \frac{1}{x} = \frac{1}{3}.\end{aligned}$$

(c) Compute $\lim_{x\to+\infty} \dfrac{3x}{\sqrt{4x^2+3}}$

Solution:

$$\begin{aligned}\lim_{x\to-\infty} \frac{3x}{\sqrt{4x^2+3}} &= \lim_{x\to-\infty} \frac{3x/x}{\sqrt{4x^2+3}/\sqrt{x^2}} \\ &= \lim_{x\to-\infty} \frac{3}{\sqrt{4+3/x^2}} = \frac{3}{2}.\end{aligned}$$

Major Topic 2: Computing Derivatives

(a) Compute the derivative of $f(x) = (2x^4+3)(5x-2\sqrt{x})$, explicitly justifying each step and naming each derivative rule you use.

Solution:

$$\begin{aligned}
f'(x) &= (2x^4+3)'(5x-2\sqrt{x}) + (2x^4+3)(5x-2\sqrt{x})' && \text{Product Rule}\\
&= \Big((2x^4)' + (3)'\Big)(5x-2\sqrt{x}) + (2x^4+3)\Big((5x)' - (2\sqrt{x})'\Big) && \text{Sum Rule}\\
&= \Big((2x^4) + 0\Big)(5x-2\sqrt{x}) + (2x^4+3)\Big((5x)' - (2\sqrt{x})'\Big) && \text{Constants Rule}\\
&= \Big(2(x^4)'\Big)(5x-2\sqrt{x}) + (2x^4+3)\Big(5(x)' - 2(\sqrt{x})'\Big) && \text{Scalar Products}\\
&= \Big(2(x^4)'\Big)(5x-2\sqrt{x}) + (2x^4+3)\Big(5\cdot 1 - 2(\sqrt{x})'\Big) && \text{Identity}\\
&= (2\cdot 4x^3)(5x-2\sqrt{x}) + (2x^4+3)\left(5\cdot 1 - 2\cdot\frac{1}{2}x^{-1/2}\right) && \text{Power Rule.}
\end{aligned}$$

(b) $\dfrac{d}{dx}\sec\left(\dfrac{x^2+1}{\sqrt{x^3-2}}\right) =$

Solution:

$$\sec\left(\frac{x^2+1}{\sqrt{x^3-2}}\right)\tan\left(\frac{x^2+1}{\sqrt{x^3-2}}\right)\frac{2x\sqrt{x^3-2} - (x^2+1)\frac{1}{2}(x^3-2)^{-1/2}\cdot 3x^2}{x^3-2}.$$

(c) Compute $\dfrac{d}{dt}\sqrt[5]{\dfrac{\tan^2(t^2+1)+2}{\sin(2t)-2t}} =$

Solution:

$$\frac{d}{dt}\sqrt[5]{\frac{\tan^2(t^2+1)+2}{\sin(2t)-2t}} = \frac{1}{5}\left(\frac{\tan^2(t^2+1)+2}{\sin(2t)-2t}\right)^{-4/5} \cdot \frac{(2\tan(t^2+1)\sec^2(t^2+1)2t)(\sin(2t)-2t) - (2\cos(2t)-2)(\tan^2(t^2+1)+2)}{(\sin(2t)-2t)^2}.$$

Secondary Topic 2: Definition of Derivative

(a) If $f(x) = \sqrt{x+3}$, find $f'(6)$, directly from the definition of derivative.

Solution:

$$\begin{aligned} f'(2) &= \lim_{h\to 0} \frac{f(6+h)-f(6)}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} \\ &= \lim_{h\to 0} \frac{h}{h(\sqrt{9+h}+3)} \\ &= \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3} \\ &= \lim_{h\to 0} \frac{1}{6}. \end{aligned}$$

(b) If $g(x) = \frac{1}{x+2}$, find $g'(a)$, explicitly using the definition of the derivative.

Solution:

$$\begin{aligned} g'(x) &= \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} \\ &= \lim_{h\to 0} \frac{(x+2)-(x+h+2)}{(x+2)(x+h+2)h} \\ &= \lim_{h\to 0} \frac{-h}{(x+2)(x+h+2)h} \\ &= \lim_{h\to 0} \frac{-1}{(x+2)(x+h+2)} = \frac{-1}{(x+2)^2}. \end{aligned}$$