Math 1231 Fall 2026
Single-Variable Calculus I Section 11
Mastery Quiz Worksheet 6
Due Friday, October 2

This week's mastery quiz has three topics. It is the first time we have seen topic S3 or S4.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, you should justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

On Friday, we will end class with a short quiz, containing questions drawn from this worksheet, plus one "easy" question on topic M1. You may not use or consult any materials or notes during Friday's quiz; make sure you work through these problems carefully, and understand them well enough that you can replicate that work in class.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Secondary Topic 3: Linear Approximation
- Secondary Topic 4: Rates of Change

Major Topic 2: Computing Derivatives

(a) Explicitly justifying each step and naming each derivative rule you use, compute $\frac{d}{dx}\frac{\sin(x)+1}{2x^2-5}$.

Solution:

$$\begin{aligned}
\frac{d}{dx}\frac{\sin(x)+1}{2x^2-5} &= \frac{(\sin(x)+1)'(2x^2-5)-(2x^2-5)'(\sin(x)+1)}{(2x^2-5)^2} && \text{Quotient rule}\\
&= \frac{\big((\sin(x))'+(1)'\big)(2x^2-5)-\big((2x^2)'-(5)'\big)(\sin(x)+1)}{(2x^2-5)^2} && \text{Sum Rule}\\
&= \frac{(\sin(x))'(2x^2-5)-(2x^2)'(\sin(x)+1)}{(2x^2-5)^2} && \text{Constants rule}\\
&= \frac{\cos(x)(2x^2-5)-(2x^2)'(\sin(x)+1)}{(2x^2-5)^2} && \text{Derivative of sine}\\
&= \frac{\cos(x)(2x^2-5)-2(x^2)'(\sin(x)+1)}{(2x^2-5)^2} && \text{Scalar Products}\\
&= \frac{\cos(x)(2x^2-5)-2(2x)(\sin(x)+1)}{(2x^2-5)^2} && \text{Power Rule.}
\end{aligned}$$

(b) $\dfrac{d}{dx}\sec(\tan(\cos((x+1)^2))) =$

Solution:

$$\sec(\tan(\cos((x+1)^2)))\tan(\tan(\cos((x+1)^2)))\sec^2(\cos((x+1)^2))(-\sin((x+1)^2))2(x+1)$$

(c) $\dfrac{d}{dx}\sec\left(\dfrac{x^2+1}{\sqrt{x^3-2}}\right) =$

Solution:

$$\sec\left(\frac{x^2+1}{\sqrt{x^3-2}}\right)\tan\left(\frac{x^2+1}{\sqrt{x^3-2}}\right)\frac{2x\sqrt{x^3-2}-(x^2+1)\frac{1}{2}(x^3-2)^{-1/2}\cdot 3x^2}{x^3-2}.$$

Secondary Topic 3: Linear Approximation

(a) Find a linear approximation to the function $f(x)=\frac{x^3}{1+x}$ near the point $a=1$ and use it to approximate $f(1.3)$.

Solution:

$$\begin{aligned} f(1) &= \frac{1}{2} \\ f'(x) &= \frac{3x^2(1+x) - x^3}{(1+x)^2} \\ f'(1) &= \frac{6-1}{4} = \frac{5}{4} \\ f(x) &\approx \frac{1}{2} + \frac{5}{4}(x-1) \\ f(1.3) &\approx \frac{1}{2} + \frac{5}{4} \cdot .3 = \frac{20}{40} + \frac{15}{40} = \frac{35}{40} = \frac{7}{8}. \end{aligned}$$

(b) Write the equation for the tangent line to $g(x) = 2x - \tan(x)$ at the point $a = \pi$.

Solution:

$$\begin{aligned} g(\pi) &= 2\pi - 0 = \pi \\ g'(x) &= 2 - \sec^2(x) \\ g'(\pi) &= 2 - 1 = 1 \\ y &= 2\pi + (x - \pi) \end{aligned}$$

Secondary Topic 4: Rates of Change

(a) Suppose that a factory produces widgets, and if p people work at the factory then they will produce a total of $W(p) = 30\sqrt{p}$ widgets.

(i) What are the units of $W'(p)$? What does it represent physically? What does it mean if W' is big?

(ii) Calculate $W'(9)$. What does this tell you physically? What physical observation could you make to check your calculation?

Solution:

(i) $W'(p)$ has units of widgets per person. It describes the rate at which the number of widgets increases as we add more people to the factory (this is called the marginal product of labor, but you don't need to know that). If it's large, that means that adding one more person to the factory will let us produce a lot more widgets.

(ii) $W'(p) = \frac{15}{\sqrt{p}}$ so $W'(9) = 5$. So moving from nine people to ten people working at the factory will lead to the production of five extra widgets.

(b) Suppose the height of a particle in centimeters is given as a function of time in seconds $p(t) = t^3 - 3t$.

(i) When is the velocity zero?

(ii) When is the acceleration zero?

Solution:

(i) $p'(t) = 3t^2 - 3$ is zero when $t = \pm 1$ second.

(ii) $p''(t) = 6t$ is zero when $t = 0$ seconds.