Math 1231: Single-Variable Calculus I
George Washington University Fall 2026
Recitation 7

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Problem 1. Consider the function $f(x) = x^3 - 3x^2 + 1$ on $[-1, 4]$.

(a) Does this function have absolute extrema? Why?

(b) What are the critical points of this function?

(c) How many absolute extrema are there? What are they, and where are they?

Problem 2. Let's find the global extrema of $g(x) = \sqrt[3]{x^3 + 6x^2}$ on the closed interval $[-5, 5]$.

(a) Does this function have absolute extrema? Why?

(b) What are the critical points of this function?

(c) How many absolute extrema are there? What are they, and where are they? (Hint: you may need to use a calculator at the last step.)

Problem 3. Let $f(x) = |x^3 - 3x|$ on the closed interval $[-1, 3]$.

(a) Does this function have absolute extrema? Why?

(b) What are the critical points of this function?

(c) How many absolute extrema are there? What are they, and where are they?

Problem 4. We can use the mean value theorem to constrain the possible values for a function. For instance, suppose I have a function f, and all I know is that $f(1) = 10$ and $f'(x) \geq 2$ for every x. I want to know about $f(4)$.

(a) What would this mean in English? How should we think about this physically? What does that tell you about $f(4)$?

(b) Use the Mean Value Theorem to set up an equation relating $f(1), f(4)$, and $f'(x)$. What does it tell you about $f(4)$?

(c) How do those two arguments relate to each other?

(d) is it possible for $f(4) = -30$?

Problem 5. Suppose $|g'(x)| \leq 2$ for all x, and $g(0) = 7$. We want to know about $g(5)$?

(a) What would this mean in English? How should we think about this physically? What does that tell you about $g(5)$?

(b) Use the Mean Value Theorem to set up an equation relating $g(0), g(5)$, and $g'(x)$. What does it tell you about $f(4)$?

(c) How do those two arguments relate to each other?

(d) is it possible for $g(5) = -30$?