Math 1231 Fall 2026
Single-Variable Calculus I Section 12
Mastery Quiz Worksheet 8
Due Friday, October 16

This week's mastery quiz has three topics. It is the last time we will see topic M2, S5, or S6. The quiz on Friday will have four questions and should take about fifteen minutes.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, you should justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

On Friday, we will end class with a short quiz, containing questions drawn from this worksheet, plus one "easy" question on topic M1. You may not use or consult any materials or notes during Friday's quiz; make sure you work through these problems carefully, and understand them well enough that you can replicate that work in class.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Secondary Topic 5: Implicit Differentiation
- Secondary Topic 6: Related Rates

Major Topic 2: Computing Derivatives

(a) Compute $\dfrac{d}{dx}\sec\left(\dfrac{x^3-x}{\sqrt[5]{x}+1}\right)=$

Solution:

$$\sec\left(\frac{x^3-x}{\sqrt[5]{x}+1}\right)\tan\left(\frac{x^3-x}{\sqrt[5]{x}+1}\right)\frac{(3x^2-1)(\sqrt[5]{x}+1)-\frac{1}{5}x^{-4/5}(x^3-x)}{(\sqrt[5]{x}+1)^2}$$

(b) Compute $\dfrac{d}{dx}\csc^{7/3}\left(\dfrac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)$.

Solution:

$$\frac{7}{3}\csc^{4/3}\left(\frac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)\left(-\csc\left(\frac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)\cot\left(\frac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)\right)$$
$$\cdot\frac{(2x+\cos(x))(\sqrt{x}-\cot(x))-\left(\frac{1}{2}x^{-1/2}+\csc^2(x)\right)(x^2+\sin(x))}{(\sqrt{x}-\cot(x))^2}.$$

(c) Compute $\dfrac{d}{dx}\tan(x^2+\csc^2(x^3+\sin(x^5)))$.

Solution:

$$\frac{d}{dx}\tan(x^2+\csc(x^3+\sin(x^5)))$$
$$=\sec^2(x^2+\csc(x^3+\sin(x^5)))\Big(2x-2\csc^2(x^3+\sin(x^5))\cot(x^3+\sin(x^5))\Big)\cdot(3x^2+\cos(x^5)\cdot 5x^4).$$

Secondary Topic 5: Implicit Differentiation

(a) Find a formula for y' in terms of x and y if $x^3y+x^2y^2+y^4=0$.

Solution:

$$\begin{aligned}3x^2y+x^3y'+2xy^2+2x^2yy'+4y^3y'&=0\\ x^3y'+2x^2yy'+4y^3y'&=-3x^2y-2xy^2\\ y'&=-\frac{3x^2y+2xy^2}{x^3+2x^2y+4y^3}.\end{aligned}$$

(b) Find a tangent line to the curve given by $x^4-2x^2y^2+y^4=16$ at the point $(\sqrt{5},1)$.

Solution: We use implicit differentiation, and find that

$$\begin{aligned} 4x^3 - 2\left((2xy^2 + x^2 2y\frac{dy}{dx}\right) + 4y^3\frac{dy}{dx} &= 0 \\ 4x^3 - 4xy^2 &= 4x^2 y\frac{dy}{dx} - 4y^3\frac{dy}{dx} \\ \frac{4x^3 - 4xy^2}{4x^2 y - 4y^3} &= \frac{dy}{dx} \end{aligned}$$

Thus at the point $(\sqrt{5}, 1)$ we have

$$\frac{dy}{dx} = \frac{4\sqrt{5}^3 - 4\sqrt{5}\cdot 1^2}{4\sqrt{5}^2\cdot 1 - 4\cdot 1^3} = \sqrt{5}\left(\frac{20-4}{20-4}\right) = \sqrt{5}.$$

Thus the equation of our tangent line is

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 1 &= \sqrt{5}(x - \sqrt{5}). \end{aligned}$$

Secondary Topic 6: Related Rates

(a) The surface area of a cube is given by the formula $A = 6s^2$ where s is the length of a side. If the side lengths are increasing by 2 inches per second, how fast is the surface area increasing when the area is 54 square inches?

Solution: We have the data $A = 6s^2, A = 54, s' = 2$. We take a derivative and see that $A' = 12ss'$, so we need to find s. But when $A = 54$ we have

$$\begin{aligned} 54 &= 6s^2 \\ 9 &= s^2 \\ 3 &= s \end{aligned}$$

and thus

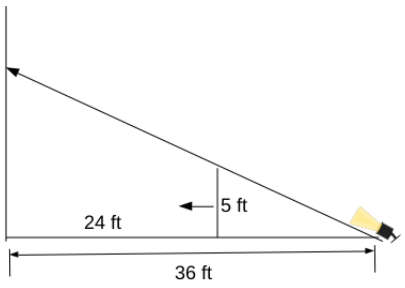
$$A' = 12ss' = 12\cdot 3\cdot 2 = 72$$

so the area is increasing at 72 square inches per second.

(b) A spot light is on the ground 36 ft away from a wall and a 5 ft tall person is walking towards the wall at a rate of 4 ft/sec. How fast is the height of the shadow changing when the person is 24 feet from the wall? Is the shadow increasing or decreasing in height at this time?

(i) Choose an equation to use for this problem, and explain why you chose that equation.

(ii) Use calculus to answer the question. Make sure you answer with a complete sentence that clearly and directly answers the question.



Solution: Let h be the height of the shadow, and d be the distance between the wall and the person. Then we want to find h'. We currently have $d = 24$. We know by similar triangles that $\frac{36-d}{36} = \frac{5}{h}$, which tells us that currently $h = 15$.

Then we have $d' = -4$. We compute

$$\begin{aligned} \frac{-d'}{36} &= \frac{-5h'}{h^2} \\ \frac{1}{9} &= \frac{-h'}{45} \\ h' &= \frac{-45}{9} = -5. \end{aligned}$$

Thus the shadow's height is decreasing by 5 feet per second.