Math 1231: Single-Variable Calculus I
George Washington University Fall 2026
Recitation 9

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Problem 1. Let $f(x) = 2x^3 + 3x^2 - 36x$.

(a) Find the critical points of f.

(b) Which of these points can you classify using the second derivative test?

(c) Classify all the critical points using the first derivative test.

Problem 2. Let $g(x) = \tan(x) - 2x$.

(a) Find the critical points of g.

(b) Which of these critical points can you classify using the second derivative test?

(c) Classify that critical points of g.

Problem 3 (Bonus)**.** We talked about using the combination of the Intermediate Value Theorem and Rolle's Theorem to figure out exactly how many solutions an equation has.

Let $h(x) = x^5 + x^3 + x - 1$. This is a function which has no roots you can actually write down in a useful way; it's a theorem that you can't give a nice algebraic description of the solutions to $x^5 + x^3 - 1 = 0$. But we can say some things about them.

(a) First, we want to get an idea of how many solutions we expect. Try plugging some small, easy numbers into this function, until you think you understand the function. How many solutions should it have?

(b) Now we want to prove we have to have a solution. Write up an argument in terms of the Intermediate Value Theorem to prove that $x^5 + x^3 - 1 = 0$ has a solution.

(c) Now we'll prove that there can't be any more. Check that this function satisfies the conditions of Rolle's Theorem. How does that prove we can't have two solutions?

(d) Can you explain informally why the equation can only have one solution, in terms of rates of change?

(e) Bonus: could you make the same argument about $h_2(x) = x^5 + x^3 - 1$? Why or why not? What would change?

Problem 4. Let $g(x) = x\tan(x)$. We want to sketch a graph of g.

(a) What is the domain of g?

(b) For simplicity, let's just look at $[-\pi/2, \pi/2]$. What can you say about any asymptotes it has?

(c) Does this function have any roots you can find?

(d) $g'(x) = \frac{\sin(x)\cos(x)+x}{\cos^2(x)}$. What are the critical points?

(Hint: when is $\sin(x)\cos(x)$ positive and when is it negative?)

(e) What are the critical values?

(f) Where is g increasing and decreasing? Does it have maxima or minima?

(g) $g''(x) = 2\sec^2(x)(1 + x\tan(x))$. Where are the potential points of inflection, and what are their values? Where is h concave up and down?

(h) Sketch the graph.

Problem 5. Let $h(x) = \frac{x+2}{x-1}$. We want to sketch a graph of h.

(a) What is the domain of h? What can you say about any asymptotes it has?

(b) Does this function have any roots? Where?

(c) What happens as x approaches $+\infty$? $-\infty$?

(d) $h'(x) = -3(x-1)^{-2}$. What are the critical points and values?

(e) Where is h increasing and decreasing? Does it have maxima or minima?

(f) $h''(x) = 6(x-1)^{-3}$. Where are the potential points of inflection? Where is h concave up and down?

(g) Sketch the graph.

Problem 6 (Bonus)**.** Let $g(x) = x^5 - 4x^3 + 4x + 7$. We want to sketch a graph of g.

(a) What is the domain of g? What can you say about any asymptotes it has?

(b) Does this function have any roots you can find?

(c) What happens as x approaches $+\infty$? $-\infty$?

(d) $g'(x) = 5x^4 - 12x^2 + 4$. What are the critical points?

(Hint: if we set $u = x^2$ this becomes a quadratic, and we can factor it.)

(e) What are the critical values?

(Hint: $g(x) = 7 + x(x^4 - 4x^2 + 4) = 7 + x(u^2 - 4u + 4)$.)

(f) Where is g increasing and decreasing? Does it have maxima or minima?

(g) $g''(x) = 20x^3 - 24x$. Where are the potential points of inflection, and what are their values? Where is h concave up and down?

(h) Sketch the graph.