Math 1231 Fall 2026
Single-Variable Calculus I Section 12
Mastery Quiz Worksheet 9
Due Friday, October 23

This week's mastery quiz has two topics. It is the first time we will see topics M3 or S7. The quiz on Friday will have three questions and should take about fifteen minutes.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, you should justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

On Friday, we will end class with a short quiz, containing questions drawn from this worksheet, plus one "easy" question on topic M1. You may not use or consult any materials or notes during Friday's quiz; make sure you work through these problems carefully, and understand them well enough that you can replicate that work in class.

Topics on This Quiz

- Major Topic 3: Optimization and Extrema
- Secondary Topic 7: Curve Sketching

Major Topic 3: Opitmization and Extrema

(a) The function $f(x) = \dfrac{x^2+5}{x+2}$ has absolute extrema either on the interval $[-3,0]$ or on the interval $[0,3]$. Pick one of those intervals, explain why f has extrema on that interval, and find the absolute extrema.

Solution: f is continuous on the closed interval $[0,3]$, so it must have extrema there. (It is *not* continuous on $[-3,0]$ because it is undefined at -2.)

We compute

$$f'(x) = \frac{2x(x+2)-(x^2+5)}{(x+2)^2} = \frac{x^2+4x-5}{(x+2)^2}$$
$$= \frac{(x+5)(x-1)}{(x+2)^2}$$

is zero for $x = 1, -5$ and is undefined for $x = -2$, so those are the critical points. The only one we have to care about is $x = 1$.

$$f(0) = 5/2$$
$$f(1) = 2$$
$$f(2) = 9/4$$

so the absolute minimum is 2 at 1, and the absolute maximum is $5/2$ at 0.

(b) Classify the critical points and relative extrema of $f(x) = 5 + 8x^3 + x^4$.

Solution: We have $f'(x) = 24x^2 + 4x^3 = 4x^2(6+x)$. So the critical points are $-6, 0$. We could try the second derivative test: we get $f''(x) = 48x + 12x^2$. Then $f''(-6) = 12(-24+36) = 144 > 0$, so this is a relative minimum. But $f''(0) = 0$ which doesn't tell us anything. We have to pass to the first derivative test.

We make a chart:

	$4x^2$	$6+x$	$f'(x)$
$x < -6$	$+$	$-$	$-$
$-6 < x < 0$	$+$	$+$	$+$
$0 < x$	$+$	$+$	$+$

Thus f is decreasing when $x < -6$ and increasing when $x > -6$, and so we see that f has a minimum at $x = -6$, and a point which is neither a maximum nor a minimum at $x = 0$,.

Secondary Topic 7: Curve Sketching

Sketch the graph of $g(x) = 3x^4 - 4x^3 - 36x^2 + 64 = (x+2)^2(3x-4)(x-4)$. We have $g'(x) = 12x^3 - 12x^2 - 72x = 12x(x-3)(x+2)$ and $g''(x) = 36x^2 - 24x - 72 = 12(3x^2 - 2x - 6)$.

You should discuss the domain, limits, critical points, intervals of increase and decrease, concavity, and possible points of inflection.

Solution:

(i) The domain is all reals.

(ii) The roots are at $x = -2, 4/3, 4$.

(iii) We have $\lim_{x\to+\infty} g(x) = \lim_{-\infty} g(x) = +\infty$.

(iv)
$$g'(x) = 12x^3 - 12x^2 - 72x = 12x(x^2 - x - 6) = 12x(x-3)(x+2).$$
This is defined everywhere, and has roots at $-2, 0, 3$.

Thus the critical points are $x = -2, 0, 3$. We compute $g(-2) = 0, g(0) = 64, g(3) = 5^2 \cdot 5 \cdot (-1) = -125$.

(v) We make a chart:

	$12x$	$(x-3)$	$(x+2)$	$g'(x)$
$x < -2$	$-$	$-$	$-$	$-$
$-2 < x < 0$	$-$	$-$	$+$	$+$
$0 < x < 3$	$+$	$-$	$+$	$-$
$3 < x$	$+$	$+$	$+$	$+$

This implies relative minima at -2 and at 3, and a relative maximum at 0.

(vi) We compute
$$g''(x) = 36x^2 - 24x - 72 = 12(3x^2 - 2x - 6)$$
which has roots
$$x = \frac{2 \pm \sqrt{4+72}}{6} = \frac{1 \pm \sqrt{19}}{3}.$$
and there are critical points at roughly -1 and $5/3$.

Plugging in some values we have

$$g''(2) = 12 \cdot (2) = 24 > 0$$
$$g''(0) = -72 < 0$$
$$g''(-2) = 12 \cdot 10 = 120 > 0.$$

Thus the function is concave up on $\left(-\infty, \frac{1-\sqrt{19}}{3}\right) \cup \left(\frac{1+\sqrt{19}}{3}, +\infty\right)$ and is concave down on $\left(\frac{1-\sqrt{19}}{3}, \frac{1+\sqrt{19}}{3}\right)$.

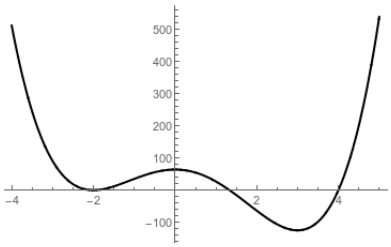

Figure 0.1: Graph of $g(x)$