Syllabus and Mathematical Reasoning

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What are we doing here?

- Math isn't just numbers and equations
  - Identify our assumptions
  - Describe them explicitly
  - Understand implications
- Mathematical ideas can explain political systems
  - How do we decide who wins elections?
  - How do we allocate political power?
  - How do we negotiate successfully?
- Approach questions like a mathematician
  - Articulate our thoughts clearly
  - Understand our own priorities
  - Converse with and persuade effectively

Boring Logistics

Textbook

The Mathematics of Politics, Second Edition

by E. Arthur Robinson and Daniel H. Ullman

https://wrlc-gwu.primo.exlibrisgroup.com/permalink/01WRLC_GWA/1j51gk4/alma99185918062204107

Course Web Page

https://jaydaigle.net/politics/

Linked from Blackboard

Contacting me

Me

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Office Hours

Office: Phillips 720E

Office hours:

Monday 2:00–5:00 PM

Wednesday 12:00 Noon–2:00 PM

Unit 1: Voting

Jan 13	Mathematical Reasoning	Jan 15	Voting Systems
Jan 20	Two-Candidate Elections	Jan 22	Criteria for Voting Systems
Jan 27	Multi-Candidate Criteria, **Quiz**	Jan 29	Theorems on Voting System Criteria
Feb 3	Evaluating Voting Systems I	Feb 5	Evaluating Voting Systems II
Feb 10	Arrow's Impossibility Theorem, **Quiz**	Feb 12	**Midterm 1**

Unit 2: Apportionment

Feb 17	The Problem of Apportionment	Feb 19	Hamilton's Method
Feb 24	Jefferson's Method	Feb 26	Divisor Methods, **Quiz**
Mar 3	Divisor Methods II	Mar 5	Evaluating Apportionment Methods
~~Mar 10~~	**Spring Break**	~~Mar 12~~	**Spring Break**
Mar 17	Apportionment and Impossibility	Mar 19	The Best Methods, **Quiz**
Mar 24	**Midterm 2**		

Unit 3: Conflict and Game Theory

Mar 26	Zero-Sum Games		
Mar 31	Strategies and Saddle Points	April 2	Probability and Randomness
April 7	Mixed Strategies, **Quiz**	April 9	Nash Equilibria
April 14	Solving 2×2 games	April 16	Conflict and Cooperation
April 21	Important Games, **Quiz**	April 23	Some Important Games

Course Structure

Grade Distribution

- Homework: 25% (drop two lowest)
- Tests: 15% each (30% total)
- Quizzes: 4% each (20% after dropping lowest)
- Final Exam: 25%

Homework

- Written or typed
- Due every Tuesday on Blackboard before class
- Must be your own work
- Two lowest scores are dropped
- For your benefit, so you can practice

Quizzes and Tests

Quiz schedule

- Six quizzes, two on each unit
- Timed, in-class
- Usually about 20 minutes
- Check your understanding of the material as we go

Test Schedule

- Test 1 on Voting: Thursday, February 12
- Test 2 on Apportionment: Tuesday, March 23
- Final exam: As scheduled by registrar

Do not book travel before finals are over!

Mathematical reasoning

1. Definitions
2. Proofs

Definitions

- Precise, not fuzzy
- Taken literally
- Not normal English meanings!
- Similar to legal reasoning

Is a bee a fish?

Each of these statutes provides that covered species include "native species or subspecies of a bird, mammal, fish, amphibian, reptile, or plant[.]" This portion of the code, however, does not elaborate on what qualifies as a bird, mammal, fish, and so forth. Based only on the qualified species listed above, bees and other land-dwelling invertebrates would not receive protection under the law. The court looked elsewhere in the Fish and Game Code for definitions to help clarify whether bees may qualify for protection under CESA.

Is a bee a fish?

Importantly, the section 45 of the code defines "fish" as "a wild fish, mollusk, crustacean, invertebrate, amphibian, or part, spawn, or ovum of any of those animals." (Emphasis added). According to the court, the term "invertebrate" under the definition of fish includes both aquatic and terrestrial invertebrates, such as bees.

Definition of a function

Definition

A function is a rule that assigns exactly one output to every valid input. We call the set of valid inputs the domain of the function, and the set of possible outputs the codomain or sometimes the range. A function must be deterministic, in that given the same input it will always yield the same output.

Definition (Unnecessarily technical definition)

Let A, B be sets. We define a function $f : A \to B$ to be a set of ordered pairs $\{(a, b) : a \in A, b \in B\}$ such that for each a in A there is exactly one pair whose first element is a. We call A the domain and B the codomain of f.

Proofs

Types of theorems

- "Theorem" for a major, important result
- "Proposition" for a less important result that we still care about
- "Lemma" for annoying technical results we mainly want in order to prove something else we actually care about
- "Corollary" for something that follows immediately from something we've already proven.

Theorems are universal

- Need universal arguments to prove something is always true
- Need one counterexample to prove something is not always true
- (Need universal arguments to prove something is never true)

Example

- All swans are white
- No swans are red
- No black swan is white

Hypotheses: When Things Break

A theorem tells you when something can break

Hypotheses are conditions that force a result to be true.

Example

If we decrease revenue and increase spending, the deficit will increase.

Example

A voting method that is anonymous and neutral cannot be decisive.

Theorems tell us about tradeoffs we can't avoid.

Discussion Question

Alex, Bailey, and Casey are running for office.

33 voters like Alex best	18 prefer Bailey to Casey
	15 prefer Casey to Bailey
32 voters like Bailey best	24 prefer Alex to Casey
	8 prefer Casey to Alex
34 voters like Casey best	16 prefer Alex to Bailey
	18 prefer Bailey to Alex

Question

Who should win?

Another view

18	15	24	8	16	18
A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

Questions to think about

- Who has the most first-place votes? C has 34
- Who has the least first-place votes? B has 32
- Who has the most *last*-place votes? C has 42
- Who has the least last-place votes? A has 26

Another view

18	15	24	8	16	18
A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

More Questions

- Who wins between A and B? B wins 50 to 49
- Who wins between A and C? A wins 57 to 42
- Who wins between B and C? B wins 50 to 49

Terminology

Consider an election with multiple candidates

Definition

The set of candidates is the slate. A, B, C, . . .

The set of voters is the electorate.

Each voter submits a preference ballot, listing candidates in descending order of preference.

B
D
C
A

Basic Assumptions

Logistics: not our problem

We assume the votes are already collected and counted

- Who gets to vote?
- How do they submit their votes?
- What if voters make mistakes?
- What if we count the votes wrong?

Important questions! But we'll ignore them.

Basic Assumptions

Rational voters

We assume voter preferences rational or transitive

If $B > D$ and $D > C$ then $B > C$.

True for numbers. Is it true for people?

Voter profiles

A	B	A	C	A	B	C
B	C	B	A	B	C	A
C	A	C	B	C	A	B

Tabulated Profiles

3	2	2
A	B	C
B	C	A
C	A	B

Social Choice Functions

Definition

A social choice function for a slate of candidates takes in a voter profile, and outputs a non-empty subset of the slate.

We think of this subset as the list of "winners" of the election. We must have at least one winner, but we can have multiple winners (which you might interpret as a tie).

- Lots of possible functions!
- With three candidates and four voters, number of possible functions is a thousand digit number.
- But most aren't very interesting.

Plurality Voting

Definition

A candidate who gets more votes than any other candidate is said to have a plurality of the votes.

In the plurality method we select as the winner the candidate who is ranked first by the largest number of voters. In the case that there is a tie for the most first-choice votes, we select all the candidates who tie for the most first-choice votes.

- The good: simple and familiar
- The bad: doesn't use much information!

Plurality Voting Example

3	2	2
A	B	C
B	C	A
C	A	B

Who wins?

A has three first-place votes, while B and C each have two. So A wins.

Plurality Voting Problems

5	4	4	4	3
A	B	C	D	E
B	C	B	B	D
C	E	D	E	B
E	D	E	C	C
D	A	A	A	A

Who wins?

A has five first-place votes, while B, C, and D each have four and E has three. So A wins.

But fifteen people hate A. Is this a good idea?