

Famous Games that Make Useful Metaphors

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Games as Metaphors

- Rarely play bimatrix games in daily life
- Useful as a framework for thinking about strategic decisions
- Want to leave you with several “standard” games that are useful metaphors for situations we encounter
- Think about the math of the game, but also when you see similar situations in your life

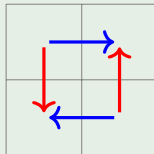
A disclaimer

- The “standard” presentations are mostly from the 1950s.
- You can tell.
- I'll mostly present them as historically given, and then talk about more modern versions.

Battle of the Sexes

Example

	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5

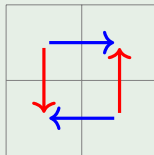


- Two saddle points: both go to boxing or both go to ballet
- Also a mixed Nash equilibrium:
 - Row goes boxing $1/4$ the time
 - Column goes to ballet $1/4$ the time.
 - Expected value for each player is 2.5.
- All of these equilibria seem bad
 - The first two are persistently unfair
 - The third is worse for both players than *either* saddle point.

Battle of the Sexes

Example

	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5

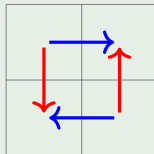


- “Reasonable” solution: compromise.
 - Take turns or flip a coin
 - Either way, expected value is 7.5.
- What if you can’t coordinate?

Battle of the Sexes

Example

	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5



- Or what if you're an asshole?
 - Loudly announce you're going to the ballet
 - Column will be better off yielding
 - Requires credible **precommitment**

Battle of the Sexes

Key Questions

- How to distribute benefits fairly
- How to get more of the pie without shrinking the pie

Example

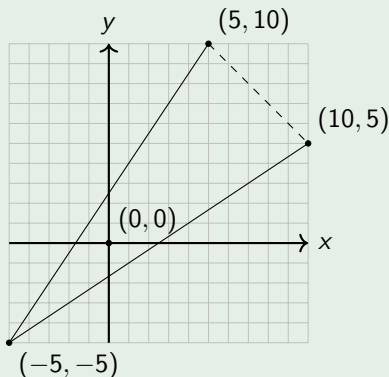
- Where to go on a date
- Pricing a deal
- Signing a treaty
- Congressional bills

The Payoff Polygon

Definition

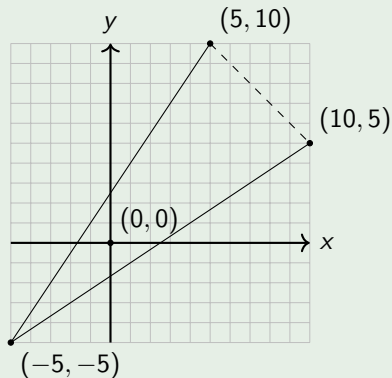
- View each pair of payoffs as a point on the plane
- Plot all these points
- Draw the smallest (convex) polygon that contains them
- We call this the **payoff polygon**

Example (Battle of the Sexes)



The Payoff Polygon

Example (Battle of the Sexes)



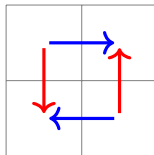
- Row wants to move right
- Column wants to move up
- The top-right boundary is **Pareto optimal** or **Pareto efficient**
 - Any line segment angled down-right
- Any other point in the polygon is attainable (with mixed strategies), but **Pareto inferior**

Chicken

Example

- Two drivers drive straight at each other
- If neither swerves, they crash and injure themselves
- But if one swerves, he's a "chicken", which is embarrassing.

	Swerve	Don't
Swerve	0, 0	-5, 10
Don't	10, -5	-10, -10

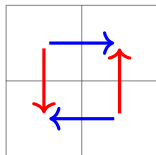


Discussion Question

- What should each player do?
- How is this different from "Battle of the Sexes"?

Chicken

	Swerve	Don't
Swerve	0, 0	-5, 10
Don't	10, -5	-10, -10



- Two saddle points
- Mixed Nash equilibrium: swerve $1/3$ of the time
 - Expected value is $-10/3$
 - Worse than your two good outcomes, but better than either bad outcome.

Chicken versus Battle of the Sexes

	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5

	Swerve	Don't
Swerve	0, 0	-5, 10
Don't	10, -5	-10, -10

	Boxing	Ballet
Ballet	3rd, 3rd	1st, 2nd
Boxing	2nd, 1st	4th, 4th

	Swerve	Don't
Swerve	2nd, 2nd	3rd, 1st
Don't	1st, 3rd	4th, 4th

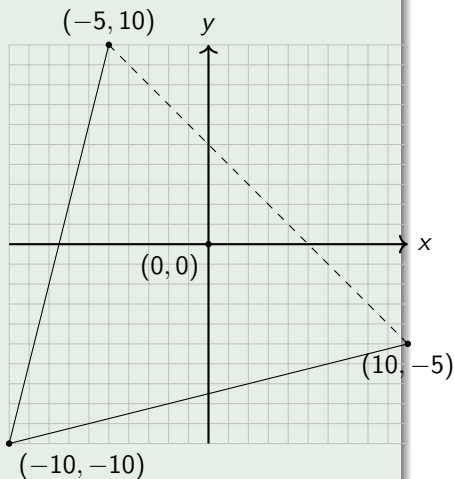
- In Battle of the Sexes you have the same top two
- In Chicken you don't.

Chicken

	Swerve	Don't
Swerve	0, 0	-5, 10
Don't	10, -5	-10, -10

- Flipping a coin would still be good for both players.

Example (Chicken)



Chicken

Key Questions

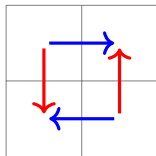
- How to save face
- How to avoid blackmail or extortion

Example

- Any sort of dare
- Threats of war
- Threats of price wars
- Message bills with bad policy
- Tit-for-tat gerrymandering

Chicken Variations

	Swerve	Don't
Swerve	0, 0	-5, 2
Don't	2, -5	-10, -10



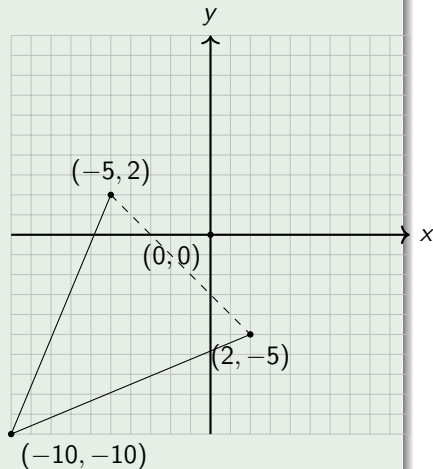
- Two saddle points
- Mixed Nash equilibrium: swerve $5/7$ of the time
 - That's more often than the $1/3$ we got last time
- Expected value is $-10/7$
 - That's *better* than before
 - Lowering payout improved the mixed Nash equilibrium!

Chicken

	Swerve	Don't
Swerve	0, 0	-5, 2
Don't	2, -5	-10, -10

- This triangle isn't actually the payoff polygon

Example (Chicken Variant)

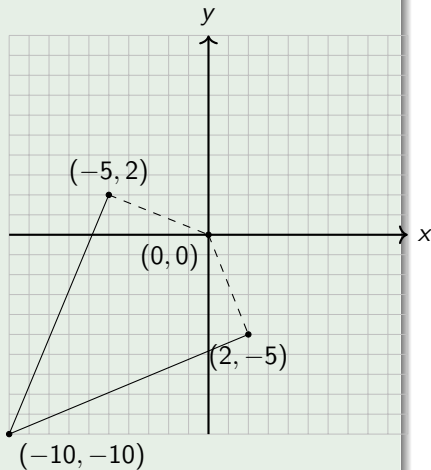


Chicken

	Swerve	Don't
Swerve	0, 0	-5, 2
Don't	2, -5	-10, -10

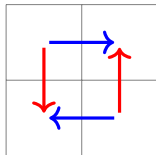
- This is the payoff polygon
- If they can coordinate, both players should just swerve.

Example (Chicken Variant)



Chicken Variations

	Swerve	Don't
Swerve	0, 0	-5, 10
Don't	10, -5	-50, -50



- Two saddle points
- Mixed Nash equilibrium: swerve $9/11$ of the time
 - Most of the time!
- Expected value is $-10/11$
 - That's *better* than either other option
 - Even though the worst case is *really* bad.
- Because a crash is so bad, both parties are incentivized to swerve.

Chicken Variations

Example

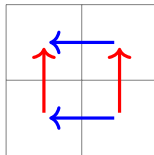
- Mutually assured destruction
 - As war gets more destructive, people try harder to avoid it
 - Modernization has maybe moved war from the first variant to the second or third variant
- Congressional poison pills
 - Make a bill worse so it doesn't pass
- "Madman theory"
 - Act so crazy that people think you won't swerve anyway
- Cuban Missile Crisis

Prisoner's Dilemma

Example

- Two criminals are arrested and interrogated separately
- If neither confesses, they get off on a minor charge
- If both confess, both are convicted of a serious charge
- If *one* confesses, he gets off free but the other is convicted with no mercy

	Confess	Don't
Confess	-5, -5	0, -10
Don't	-10, 0	-2, -2

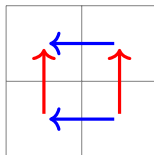


Discussion Question

- What should each player do?

Prisoner's Dilemma

	Confess	Don't
Confess	-5, -5	0, -10
Don't	-10, 0	-2, -2



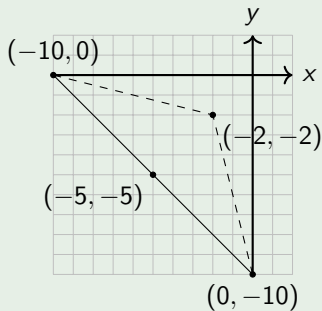
- Unique saddle point where both confess
- In fact, "Confess" dominates "Don't Confess"
- Simple game, right?

Prisoner's Dilemma

	Confess	Don't
Confess	-5, -5	0, -10
Don't	-10, 0	-2, -2

- Best outcome overall is neither confessing
- But that's *not* a Nash equilibrium.

Example (Prisoner's Dilemma)



Prisoner's Dilemma

- Maybe a good thing it's hard for criminals to coordinate?
- But these coordination problems show up everywhere.

	Confess	Don't
Confess	-5, -5	0, -10
Don't	-10, 0	-2, -2

	Defect	Cooperate
Defect	-5, -5	0, -10
Cooperate	-10, 0	-2, -2

Example

- Climate change
- Cleaning common spaces
- "Public goods"
- Arms Races
- Cartels
- Advertising
- Sports doping
- Large Cars

Prisoner's Dilemma

- The *two players* are better off cooperating
- May or may not be good for everyone else!
- Sometimes the public policy goal is to get people to cooperate
- Other times the goal is to prevent cooperation *against* society.

Penalize defectors

- Pass a law taxing pollution
- Sports leagues test for and ban doping
- Gangs can threaten snitches with violence

Subsidize cooperation

- Subsidies for clean energy
- Publicly funded cleanup efforts

Repeated Games

- A lot of our discussion has assumed players play once, without coordination
- In the real world that's not true
- I *know* if my roommate cleans the kitchen
- This opens up different types of strategies

Tit for Tat

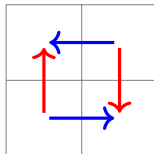
- Cooperate on turn 1
- Afterwards, play what your opponent did last time.
- Empirically does quite well

Stag Hunt

Example

- Dates to Jean-Jacques Rousseau in 1755
- Two hunters choose to hunt a stag or a hare
- Can only catch the stag with help
- Can catch a hare on your own, but it's not as good

	Hare	Stag
Hare	5, 5	8, 1
Stag	1, 8	10, 10

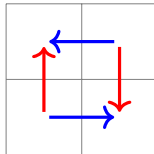


Discussion Question

- What should each player do?

Stag Hunt

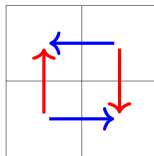
	Hare	Stag
Hare	5, 5	8, 1
Stag	1, 8	10, 10



- Two saddle points
- Mixed Nash equilibrium plays Stag $\frac{2}{3}$ of the time
 - Expected value is 7
 - Better than always-Hare, worse than always-Stag

Stag Hunt

	Hare	Stag
Hare	5, 5	8, 1
Stag	1, 8	10, 10



Discussion Question

What is the prudent strategy?

- Hare has guarantee of 5
- Stag has guarantee of 1
- Hare is prudent.
- This is a problem of coordination

Stag Hunt versus Prisoner's Dilemma

	Defect	Coop
Defect	-5, -5	0, -10
Coop	-10, 0	-2, -2

	Hare	Stag
Hare	5, 5	8, 1
Stag	1, 8	10, 10

	Defect	Coop
Defect	3rd, 3rd	1st, 4th
Coop	4th, 1st	2nd, 2nd

	Hare	Stag
Hare	3rd, 3rd	2nd, 4th
Stag	4th, 2nd	1st, 1st

- In prisoner's dilemma, cooperation is unstable
- In a stag hunt, it's a stable equilibrium
 - (But if you expect your partner to defect, it's rational to defect as well.)