# Two-Candidate Elections

Jay Daigle

jaydaigle@gwu.edu

https://jaydaigle.net/politics

The George Washington University

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# Two-Candidate Elections

**Definition**

A social choice function for two candidates is a function whose domain is the set of all possible preferences that voters could have, and whose codomain is a set with three options: $A$ wins; $B$ wins; or $A$ and $B$ tie.

- Tempting to not allow ties
- We'll see that doesn't really work.

# Voter Profiles

## Voter Profile

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| A | B | A | B | A | A | A | A | B | B |
| B | A | B | A | B | B | B | B | A | A |

## Voter Profile

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| A | B | A | B | A | A | A | A | B | B |

## Tabulated Profile

| | |
|---|---|
| 6 | 4 |
| A | B |
| B | A |

# Simple Majority

**Definition**

The simple majority method for a two-candidate election is the social choice function that selects as the winner the candidate who gets more than half of all the votes cast. If each candidate gets exactly half of the votes, then the result is a tie.

| 6 | 4 |
|---|---|
| A | B |

A wins.

# Weighted Voting

## Definition

A weighted voting method works as follows.

- There are $n$ voters, and each voter is assigned a positive number of votes.
- We will say that voter number $i$ has $w_i$ votes, or a weight of $w_i$.
- Set $t = w_1 + w_2 + \cdots + w_n$ be the total number of votes.
- A candidate who gets more than $t/2$ votes is the winner. If no candidate gets more than $t/2$ votes, the result is a tie.

# Weighted Voting Example

## Unweighted Vote

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| A | B | A | B | A | A | A | A | B | B |

A wins 6-4

## Weighted Vote

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 5 | 5 | 5 | 1 | 1 | 1 | 3 | 3 | 3 |
| A | B | A | B | A | A | A | A | B | B |

- $t = 5 + 5 + 5 + 5 + 1 + 1 + 1 + 3 + 3 + 3 = 32$
- A gets $5 + 5 + 1 + 1 + 1 + 3 = 16$
- B gets $5 + 5 + 3 + 3 = 16$
- This is a tie!

# The Dictatorship Method

**Definition**

In the dictatorship method, one of the voters is the dictator.

Whoever the dictator prefers is the winner.

**Discussion Question**

When does the dictatorship method make sense?

**Example**

Suppose we have a weighted voting method with ten voters, but the first voter gets twelve votes and each other voter gets one vote.

Then the preference of the first voter will always win the election.

**Example**

The company Meta has two classes of stock: Class A shares get one vote per share, and Class B shares get 10 votes per share.

# Supermajority Methods

## Definition

- Let $p$ be a number such that $1/2 < p \leq 1$. The supermajority method with parameter $p$ selects as the winner the candidate who gets a fraction $p$ or more of all the votes.
- If there are $t$ votes in total, a candidate must get at least $p \cdot t$ votes to win.
- If no candidate gets $p \cdot t$ votes, the result is a tie.

## Example

| | |
|---|---|
| 6 | 4 |
| A | B |

- Take $p = 2/3$
- Need $2/3 \cdot 10 = 6\frac{2}{3}$ votes to win
- This is a tie.

# Supermajority Methods

- Number of votes needed is $q$
- Call $q$ the quota
- Formula for $q$? $q \neq pt$
- $q$ is smallest *integer* as large as $pt$
- $q = \lceil pt \rceil$

**Discussion Question**

When does a supermajority method make sense?

**Poll Question**

Why don't we allow $p < 1/2$?

# Status Quo Methods

## Definition

- Start with some social choice function (such as the simple majority method or the supermajority method), which we will call the "base method".
- A status quo method designates one of the two candidates as the status quo, and the other as the challenger.
- If either candidate wins under the base method, then that candidate also wins under the status quo method.
- If there is a tie under the base method, then the status quo method names the status quo candidate as a winner.

# Status Quo Methods

## Example

| | |
|---|---|
| 6 | 4 |
| A | B |

- Supermajority with $p = 2/3$, B is status quo
- Supermajority calls this a tie
- B wins as the status quo candidate.

## Discussion Question

When does this method get used regularly? When is it a good idea?

# The Monarchy Method

## Definition

In the monarchy method, one of the candidates is a monarch.

That candidate wins regardless of how anybody votes.

## Remark

This is a constant function

Returns same output for any input.

## Poll Question

Is monarchy the same as dictatorship?

(Remember to use *our definitions*.)

## Discussion Question

When does the monarchy method make sense?

# Bloc Voting

## Definition

- A block voting method partitions the electorate into $n$ blocs, so that every voter is in exactly one bloc. Then each bloc $i$ is assigned a positive number $w_i$ of votes.
- Each bloc conducts an election using the simple majority method, possibly with some method for resolving ties. Then the bloc casts all of its $w_i$ votes in the main election for the candidate that won that simple majority election.
- The winner is the candidate who receives the most votes in the main election, with a tie if both candidates receive the same number of votes.

# Bloc Voting

## Example

| A | B | A | B | A | B | A | B | B | A | A | A | B | A | B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

- Five blocs of three voters each
- A wins blocs 1 and 4 B wins 2, 3, and 5
- B wins 3 to 2.

## Tabulated profile

| 8 | 7 |
|---|---|
| A | B |

- Simple majority: A wins
- Can we tell the bloc voting winner?

# Bloc Voting

**Discussion Question**

When would bloc voting make sense?

**Discussion Question**

Where does bloc voting get used in real life?

# Bad Ideas

- Dictatorship method
- Monarchy method

**Definition**

In the all-ties method, the election is a tie, no matter how the electorate votes.

**Definition**

In the parity method, if just one candidate gets an even number of votes, then that candidate wins. If both candidates get an odd number of votes, or both candidates get an even number of votes, then the result is a tie.

**Discussion Question**

Can we explain *why* each method is bad?

# Parity Method Example

## Example

| | |
|---|---|
| 7 | 4 |
| A | B |

- B has an even number of votes
- A has an odd number
- B wins.

## Example

What if one voter switches from B to A?

| | |
|---|---|
| 6 | 5 |
| A | B |

- Now A has an even number
- B has an odd number
- Now A wins.

# Evaluating Voting Methods

- We've seen a lot of voting methods
- There are lots more
- How do we decide what we want?

**Discussion Question**

What features should a good voting system have?

**Discussion Question**

What kind of election were you thinking about when you answered the last question?

# Anonymity

**Definition**

A method satisfies the anonymity condition (or is anonymous) if it treats all voters equally.

An anonymous method will always give the same result if the voters exchange ballots among themselves.

**Proposition**

*A method is anonymous if and only if its outcomes depend only on the tabulated profile.*

**Proof.**

See next class □

# Neutrality

**Definition**

A method satisfies the neutrality criterion or is neutral if it treats both candidates equally.

**Example**

- Neutral: Majority, supermajority, weighted voting methods
- Also neutral: dictatorship, parity, all-ties
- Not neutral: Status Quo, Monarchy

# Monotonicity

## Discussion Question

- Parity method is anonymous and neutral, but obviously bad.
- What is the problem with parity? What do we want that parity doesn't give us?