Criteria for Voting Systems

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Anonymity

Definition

- A method satisfies the anonymity condition (or is anonymous) if it treats all voters equally.
- An anonymous method will always give the same result if the voters exchange ballots among themselves.

Proposition

A method is anonymous if and only if its outcomes depend only on the tabulated profile.

Proof.

- "If and only if": We need to prove two separate things.

Proposition

A method is anonymous if and only if its outcomes depend only on the tabulated profile.

Proof.

- Suppose we have an anonymous method. Want to show two different profiles with the same tabulated profile will have the same result.
- Since two profiles give the same tabulated profile, same number of candidates prefer A in each profile. Can swap ballots among voters to get from first profile to second profile.
- Since the method is anonymous, swapping ballots can't change the result. So both profiles must have the same result.

Proposition

A method is anonymous if and only if its outcomes depend only on the tabulated profile.

Proof.

- Conversely, suppose we have a method that only depends on the tabulated profile.
- If voters swap ballots, that won't change the tabulated profile, so it won't change the result.
- So swapping ballots can't change the result, and the method is anonymous.

□

Neutrality

Definition

A method satisfies the neutrality criterion or is neutral if it treats both candidates equally.

Example

- Neutral: Majority, supermajority, weighted voting methods
- Also neutral: dictatorship, parity, all-ties
- Not neutral: Status Quo, Monarchy

Monotonicity

Discussion Question

- Parity method is anonymous and neutral, but obviously bad.
- What is the problem with parity? What do we want that parity doesn't give us?

Definition

- A method satisfies the monotonicity criterion or is monotone if a candidate is never hurt by getting more votes.
- That is: suppose the votes are cast and the method selects one candidate as the winner. Then suppose one or more voters change their votes from the losing candidate to the winning candidate. The candidate who was the winner before the change must remain the winner after the change.

The Problem with Parity

Proposition

The parity method is not monotone. (Or "violates monotonicity".)

Remark

- Monotone: something *always* happens
- Not monotone: something *doesn't always* happen
- (Not the same as "never happens"! Compare "it doesn't always rain" and "it never rains".)
- Just need one counterexample.

Proof.

A	B	A	B	A	A	A	A	B

A gets 6 votes and B gets 3 votes, so A wins.

The Problem with All-Ties

Discussion Question

- The all-ties method is anonymous, neutral, and monotone.
- Why is all-ties monotone?
- What's the problem with all-ties?

Definition

A method satisfies the decisiveness criterion or is decisive if it always chooses a winner, that is, never produces a tie.

Poll Question

Which of the methods we've discussed so far are decisive?

The Problem with Decisiveness

Discussion Question

A	B	A	B	B	A	B	A

- Who *should* win?
- Don't reference a specific voting method—what would we like to see happen?

Definition

A method satisfies the near decisiveness criterion or is nearly decisive if ties can only occur when both candidates receive the same number of votes.

Near Decisiveness

Definition

A method satisfies the near decisiveness criterion or is nearly decisive if ties can only occur when both candidates receive the same number of votes.

Remark

- Not exclusive.
- *Any* decisive method is nearly decisive.
- Not every nearly decisive method is decisive.

Simple Majority Method

Proposition

The simple majority method is nearly decisive.

Proof.

- Suppose the number t of voters is odd.
- No candidate can receive exactly half the votes, since $t/2$ is not an integer.
- So one must receive more than half; they win a majority and win the election.

Simple Majority Method

Proposition

The simple majority method is nearly decisive.

Proof.

- Now suppose t is even.
- If both candidates get $t/2$ votes, they receive the same number of votes, and tie.
- If not, one gets more than $t/2$ and thus has a majority and wins.
- So ties only occur when each candidate gets exactly $t/2$ votes.

□

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May's Theorem

Theorem (May's Theorem)

In two-candidate election, the only anonymous, neutral, monotone, and nearly decisive method is the simple majority method.

Proof.

- Suppose we have an anonymous, neutral, monotone, nearly decisive social choice function for two candidates.
- Anonymous so we only need to consider tabulated profiles
- Suppose a voters support candidate A, and b voters support candidate B. Set $t = a + b$ the total number of voters.
- Want to show that the method we are imagining must be the simple majority method. (Can't assume it's majority method!)

May's Theorem

Theorem (May's Theorem)

In two-candidate election, the only anonymous, neutral, monotone, and nearly decisive method is the simple majority method.

Proof.

- Suppose t is even.
- If $a = b = t/2$ neutrality implies we have a tie, as in Simple Majority.
- Suppose A has a majority, $a > t/2$. Want to show A wins.
- Can't assume majority wins; that's what we want to prove.
- Want to show A must win for any neutral, monotone, nearly decisive method.

Proving May's Theorem

Claim

Assume a voting method is neutral, monotone, nearly decisive. If t is even and $a > t/2$ then A has to win.

Proof.

- Since $a > t/2$, we know $a \neq b$. By nearly decisive, not a tie.
- Want to show B can't win. So think about what would happen B wins with $b = t - a$ votes.
- Since $b < t/2$, then B would win with $t/2$ votes by monotone.
- But we showed that if $a = b = t/2$ the election is a tie. So that can't be true.
- Election isn't a tie, and B doesn't win, so A wins.

Proving May's Theorem

What have we shown so far?

- Anonymous: only consider tabulated profiles.
- If t is even:
  - If $a = t/2 = b$ then the election is a tie. ✓
  - If $a > t/2$ then A wins. ✓
  - If $a < t/2$ then $b > t/2$, so B wins by neutrality. ✓
- If t is even, the method must be simple majority.

What next?

What if t is odd?

Proving May's Theorem

Claim

Assume a voting method is neutral, monotone, nearly decisive. If t is odd then the result is the same as the Simple Majority Method.

Proof.

- Neither candidate can get $t/2$ votes
- By near decisiveness, someone must win.
- Suppose A gets $a > t/2$ votes. Want to show A has to win.

Proving May's Theorem

Claim

Assume a voting method is neutral, monotone, nearly decisive. If t is odd and $a > t/2$ then A wins.

Proof.

- By near decisiveness, can't be a tie.
- We claim B can't win.
- B gets $b = t - a < t/2$ votes, so $b < a$.
- If B wins with $b < a$ votes, then B would also win with a votes by monotonicity.
- But then by neutrality A would win with a votes.

Proving May's Theorem

Claim

Assume a voting method is neutral, monotone, nearly decisive. If t is odd and $a > t/2$ then A wins.

Proof.

- By near decisiveness, can't be a tie.
- Just showed B can't win.
- So A wins.

- Showed that if $a > t/2$ then A wins. ✓
- By neutrality, if $b > t/2$ then B wins. ✓
- So if t is odd, the results match Simple Majority.

Theorem (May's Theorem)

In two-candidate election, the only anonymous, neutral, monotone, and nearly decisive method is the simple majority method.

Proof.

- By Anonymity, can just look at vote counts.
- If $a = b$, tie by Neutrality.
- If $a > t/2$:
  - Can't be a tie, by Near Decisiveness
  - B can't win, by Neutrality & Monotonicity
  - So A wins.
- If $b > t/2$, B wins by Neutrality.
- So this is precisely the Simple Majority method.

□

What does May's Theorem mean?

Theorem (May's Theorem)

In two-candidate election, the only anonymous, neutral, monotone, and nearly decisive method is the simple majority method.

- Simple Majority is anonymous, neutral, monotone, nearly decisive
- No other method we've talked about is all four
- Better than that: we *cannot* find another method that is all four.
- In a real sense, Simple Majority is the "best" method for a two-candidate race.

An Impossibility Result

Corollary

It is impossible for a voting system with two candidates to be anonymous, neutral, monotone, and decisive.

Proof.

- If it's decisive, then it's nearly decisive.
- Anonymous, neutral, monotone, and nearly decisive, must be Simple Majority
- But Simple Majority isn't decisive.
- So this is impossible.

□

An exercise for you

Theorem

In an election with two candidates, a voting method that is anonymous, neutral, and monotone must be the simple majority method, a supermajority method, or the all-ties method.

Proof.

- Think about how you'd prove this.
- Similar outline as proof of May's Theorem.

□

Back to Multi-Candidate Elections

Discussion Question

- What do we want out of a multi-candidate election?
- Which of these criteria make sense?
- What other criteria might we want?

Multi-Candidate Election methods

Multi-Candidate Voting Systems

- Plurality
- Hare's method (Instant Runoff Voting)
- Coombs's Method
- Borda Count
- Copeland's Method

Discussion Question

- How do we decide which of these are good?
- What do we want out of a multi-candidate election?