Evaluating Voting Methods I

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January 29, 2026

Summary

Voting Methods

- Plurality
- dictatorship
- Copeland's Method
- Hare's method
- monarchy
- Borda count
- Coombs's Method
- all ties
- More to come!

Voting Method Criteria

- unanimous
- anonymous
- Pareto
- Condorcet
- decisive
- neutral
- independent
- anti-Condorcet
- majoritarian
- monotone

Some Theorems

Proposition

Any social choice function that satisfies anonymity and neutrality must violate decisiveness.

Proposition (Taylor)

No social choice function involving at least three candidates satisfies both independence and the Condorcet criterion.

Proposition

If a method is Condorcet then it satisfies the majority criterion.

Analyzing the Plurality Method

Proposition

The plurality method is majoritarian, monotone, and Pareto, but not Condorcet, anti-Condorcet, or independent.

Proof.

- The majority is always a plurality, so candidate with majority will win.
- Monotone: raising a candidate on some preference lists can't reduce their first-place votes, or increase anyone else's
- Pareto: if A is ahead of B on every preference list, then B gets no votes, and can't win.

Analyzing the Plurality Method

Claim

The plurality method is neither Condorcet not anti-Condorcet.

Proof.

2	3	2
A	B	C
C	A	A
B	C	B

- B wins plurality, but
- A beats B, A beats C, and C beats B.
- A is Condorcet, and loses
- B is anti-Condorcet, and wins

□

Analyzing the Plurality Method

Claim

The plurality method is not independent.

Proof.

A	A	A	A	B	B	B
B	B	C	C	A	A	A
C	C	B	B	C	C	C

$\rightarrow$

A	A	C	C	B	B	B
B	B	A	A	A	A	A
C	C	B	B	C	C	C

- First profile: A wins, B and C lose
- Second profile: B wins, A and C lose
- But relative position of A and B doesn't change.

The Antiplurality Method

Definition

The *antiplurality method* names as winner the candidate with the fewest last-place votes.

Example

5	4	4	4	3
A	B	C	D	E
B	C	B	B	D
C	E	D	E	B
E	D	E	C	C
D	A	A	A	A

B, C, E all win.

5	4	4	4	3
B	C	A	D	E
C	A	B	A	A
E	B	E	B	B
D	E	D	E	D
A	D	C	C	C

B and E both win.

The Antiplurality Method

Definition

The *antiplurality method* names as winner the candidate with the fewest last-place votes.

Poll Question

Which criteria does the antiplurality method satisfy?

Proposition

The antiplurality method is monotone, but not majoritarian, Condorcet, anti-Condorcet, Pareto, or independent.

The Antiplurality Method

Proposition

The antiplurality method is monotone, but not majoritarian, Condorcet, anti-Condorcet, Pareto, or independent.

Claim

The antiplurality method is monotone.

Proof.

- Raising a candidate in preference lists:
  - Can't increase their last-place votes
  - Can't decrease anyone else's last-place votes.
- If a candidate wins before getting raised, they win after.

□

The Antiplurality Method

Claim

The antiplurality method is not majoritarian, Condorcet, or anti-Condorcet.

Proof.

Consider the profile:

C	C	B	B	B
A	A	C	A	A
B	B	A	C	C

- What happens? A wins.
- B gets a majority but loses. Not majoritarian.
- B is the Condorcet winner. Not Condorcet.
- A is anti-Condorcet candidate. Not anti-Condorcet.

□

The Antiplurality Method

Claim

The antiplurality method is not Pareto.

- This is surprising!
- Just *barely* true. But true.

Proof.

Consider the profile:

A	A	A
B	B	B
C	C	C

- What happens? A and B both win.
- Every voter prefers A to B, but B wins.
- Not Pareto.

□

The Antiplurality Method

Claim

The antiplurality method is not independent.

Proof.

Consider:

C	A	A	B	B
A	B	B	A	A
B	C	C	C	C

C	A	A	B	B
A	B	B	C	C
B	C	C	A	A

- What happens?
  - A wins profile 1
  - B wins profile 2
- No voter has changed preferences between A and B.
- Not independent.

□

Hare's Method

Definition

Eliminate the candidate(s) who have the fewest first-place votes.

Repeat. The last remaining candidate(s) are the winner(s).

- Popular and widely used
- Australia, New Guinea, Alaska, Maine, NYC
- Surprisingly *bad* at our criteria.

Proposition

Hare's method is majoritarian and Pareto, but not monotone, Condorcet, anti-Condorcet, or independent.

Hare's Method

Claim

Hare's method is majoritarian.

Proof.

- A majority of first-place votes will never be the fewest
- They will never be eliminated, and everyone else will.

□

Claim

Hare's method is Pareto.

Proof.

- If everyone prefers A to B, then B has no first-place votes
- B gets eliminated in the first round, and can't win.

□

Hare's Method

Claim

Hare's method is not monotone.

Proof.

6	5	4	2
A	C	B	B
B	A	C	A
C	B	A	C

$\rightarrow$

6	5	4	2
A	C	B	A
B	A	C	B
C	B	A	C

- Profile 1: Eliminate C, then B. A wins.
- Profile 2: Eliminate B, then A. C wins.
- Getting more votes makes A lose.

Hare's Method

Claim

Hare's method is not Condorcet or anti-Condorcet.

Proof.

Consider:

2	3	2
A	B	C
C	A	A
B	C	B

- What happens?
  - Eliminate A and C; B wins.
- B is the anti-Condorcet candidate and wins
- A is the Condorcet candidate and loses.

□

Hare's Method

Claim

Hare's method is not independent.

Proof.

2	2	1
B	A	A
A	C	B
C	B	C

$\rightarrow$

2	2	1
B	C	A
A	A	B
C	B	C

- Profile 1: eliminate C, then B. A wins.
- Profile 2: eliminate A, then C. B wins.
- Relative preference of A and B has not changed