Evaluating Voting Methods III

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Summary

Voting Methods

- Plurality
- dictatorship
- Copeland's Method
- Hare's method
- monarchy
- Borda count
- Coombs's Method
- all ties
- Antiplurality

Voting Method Criteria

- unanimous
- anonymous
- Pareto
- Condorcet
- decisive
- neutral
- independent
- anti-Condorcet
- majoritarian
- monotone

Previous Results

Proposition

The plurality method is majoritarian, monotone, and Pareto, but not Condorcet, anti-Condorcet, or independent.

Proposition

The antiplurality method is monotone, but not majoritarian, Condorcet, anti-Condorcet, Pareto, or independent.

Proposition

Hare's method is majoritarian and Pareto, but not monotone, Condorcet, anti-Condorcet, or independent.

Previous Results

Proposition

Coombs's method is Pareto, but not majoritarian, monotone, Condorcet, anti-Condorcet, or independent.

Proposition

The Borda count method is monotone, anti-Condorcet, and Pareto, but not majoritarian, Condorcet, or independent.

Proposition

Copeland's method is majoritarian, Condorcet, anti-Condorcet, monotone, and Pareto.

Discussion Question

Is Copeland's method independent?

Copeland's method

Claim

Copeland's method is monotone.

Proof.

- Moving A up on some lists won't hurt them in any head-to-head, so won't reduce A's score
- Won't affect any other head-to-head at all
- So it can't increase any other candidate's score
- If A wins before the switch, will also win after.

□

Discussion Question

How does this suggest we look at independence?

Copeland's method

Claim

Copeland's method is not independent.

Proof.

B	A	A
C	B	C
A	C	B

$\rightarrow$

B	A	C
C	B	A
A	C	B

- Profile 1: A gets 2, B gets 1, C gets 0. A wins.
- Profile 2: A gets 1, B gets 1, C gets 1. All candidates win
- A and B haven't changed relative positions
- B loses in profile 1 and wins in profile 2

Copeland's Method

Proposition

Copeland's method is majoritarian, Condorcet, anti-Condorcet, monotone, and Pareto, but not independent.

Black's method

- Duncan Black (1908-1991)
- *The Theory of Committees and Elections* (1958)

Definition

Black's method is the social choice function that chooses the Condorcet candidate as the unique winner if there is a Condorcet candidate, and chooses the Borda count winner if there is not.

- Attempt to combine benefits of multiple methods
- Start with Borda count, "fix" the "problem" that it's not Condorcet.

Discussion Question

What criteria will Black's method satisfy?

Black's method

Claim

Black's method is Condorcet and majoritarian.

Proof.

- Condorcet by definition.
- Majoritarian because Condorcet.

□

Claim

Black's method is anti-Condorcet

Proof.

- anti-Condorcet candidate isn't Condorcet
- anti-Condorcet candidate can't win Borda count

□

Black's Method

Claim

Black's method is monotone.

Proof.

- Two cases
- If A is Condorcet winner, moving them up in some rankings won't change that, so they still win.
- If no Condorcet winner and A wins by Borda count:
  - Moving A up can't lower their score or raise anyone else's score, so they win Borda count
  - Moving A up can't make anyone *else* into Condorcet winner
  - Moving A up could make A the Condorcet winner, but that's fine.

□

Black's Method

Claim

Black's method is Pareto.

Proof.

- Suppose all voters prefer A to B
- B is not the Condorcet candidate since they lose to A
- B can't win the Borda count because A will have more points
- B can't win.

□

Black's Method

Claim

Black's method is not independent.

Proof.

- Easy answer: it's Condorcet, so not independent. Or:

B	A	A		B	A	C
C	B	C	$\rightarrow$	C	B	A
A	C	B		A	C	B

- Profile 1: A is the Condorcet candidate, and wins
- Profile 2: no Condorcet candidate
  - Each candidate gets 3 Borda points
  - All three are winners

Black's Method

Proposition

Black's method is majoritarian, Condorcet, anti-Condorcet, monotone, and Pareto, but not independent.

Tradeoffs

Poll Question

- Which properties have been common?
- Which have been uncommon?

Discussion Question

- Why is independence so uncommon?
- Do any methods achieve independence?

Dictatorship

Definition

One voter is the dictator. Their first choice is the unique winner.

Discussion Question

What criteria does the dictatorship method satisfy?

Dictatorship

Claim

The dictatorship method is monotone.

Proof.

- If A wins, they're at the top of the dictator's preference list
- Moving them up on other lists won't change that.

□

Dictatorship

Claim

The dictatorship method is Pareto.

Proof.

- If A is higher than B on every list, then A is higher on the dictator's list
- B isn't at the top of the dictator's list, and can't win.

□

Dictatorship

Claim

The dictatorship method is independent.

Proof.

- Suppose A wins and B loses in profile 1.
- Then A is at the top of the dictator's preference list in profile 1.
- If profile 2 has A and B in the same relative positions, then B is not at the top of the dictator's preference list in profile 2.
- B can't win in profile 2.

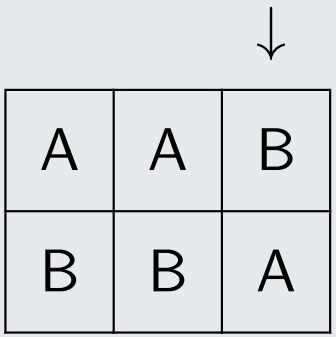
Dictatorship

Claim

The dictatorship method is not Condorcet, anti-Condorcet, or majoritarian.

Proof.

Consider:



- What happens?
- B wins
- A is the Condorcet candidate
- A is the majority candidate
- B is the anti-Condorcet candidate.

□

Dictatorship

Proposition

The dictatorship method is monotone, Pareto, and independent, but not Condorcet, anti-Condorcet, or majoritarian.

Constant Functions

Proposition

The all-ties method and the monarchy method are monotone and independent, but not Condorcet, anti-Condorcet, majority, or Pareto.

Proof.

- Constant functions: same output for any input
- Monotone and independent, because no candidate can win in one profile but lose in another
- Violate Condorcet, anti-Condorcet, majority, and Pareto, because rankings have no effect on who wins.

□

Summary

	anon	neu	unan	dec	maj	Con	AC	mono	Par	ind
Plurality	Y	Y	Y	N	Y	N	N	Y	Y	N
Antiplur	Y	Y	Y	N	N	N	N	Y	N	N
Borda	Y	Y	Y	N	N	N	Y	Y	Y	N
Hare	Y	Y	Y	N	Y	N	N	N	Y	N
Coombs	Y	Y	Y	N	N	N	N	N	Y	N
Copeland	Y	Y	Y	N	Y	Y	Y	Y	Y	N
Black	Y	Y	Y	N	Y	Y	Y	Y	Y	N
Dictator	N	Y	Y	Y	N	N	N	Y	Y	Y
All-ties	Y	Y	N	N	N	N	N	Y	N	Y
Monarchy	Y	N	N	Y	N	N	N	Y	N	Y

Tradeoffs

Proposition (Taylor)

No social choice function involving at least three candidates satisfies both independence and the Condorcet criterion.

- Condorcet isn't *that* common

Discussion Question

Why is independence so hard?

The Condorcet Paradox

Example

A	C	B
B	A	C
C	B	A

- Who should win?

Definition

- A Condorcet Paradox occurs when every candidate loses to at least one other candidate in a simple-majority head-to-head matchup.
- In this case we will get a cycle of candidates such that each beats the next head-to-head, until the last beats the first.

The Condorcet Paradox

Example

146	145	144
D	C	B
E	D	C
F	E	D
G	F	E
A	G	F
B	A	G
C	B	A

- Suppose we start with policy D
- Propose switch to C
- Passes with 289 votes
- Switch to B passes with 290 votes
- Switch to A passes with 291 votes
- Everyone is less happy!

Tradeoffs

Proposition (Taylor)

No social choice function involving at least three candidates satisfies both independence and the Condorcet criterion.

Discussion Question

Condorcet isn't that common. Why is independence so hard?

Theorem (Arrow's Impossibility Theorem)

If a social choice function with at least three candidates satisfies both Pareto and independence, then it must be a dictatorship.

Outline of the Proof

Lemma (decisiveness lemma)

A social choice function with at least three candidates that satisfies Pareto and independence must be decisive.

Lemma

Suppose a social choice function with at least three candidates satisfies Pareto and independence. Suppose there are two profiles in which no voter changes their mind about whether candidate A is preferred to candidate B. If A wins in the first profile, then B cannot win in the second profile.

Discussion Question

How is this different from independence alone?

Outline of the Proof

- Suppose we have a social choice function with more than three candidates that is Pareto and independent.
- It must be decisive.
- It satisfies the "super independence" property.

Claim

For each pair of candidates, there is some single voter who can force one candidate to lose by ranking the other candidate higher.

Claim

This voter must be the same for each pair of candidates.