Problem 1. (a) (2 points) Explain what it means for a voting method to be Pareto. (Be precise!)

Solution: If every voter prefers A to B, then B cannot win.

(b) (3 points) Explain why Hare's method is Pareto.

Solution: Suppose every voter prefers A to B. Then B gets no first-place votes, and is eliminated in the first round of voting. Thus B cannot win.

(c) (2 points) Explain what it means for a voting method to be Condorcet. (Be precise!)

Solution: A method is Condorcet if, whenever there's a Condorcet candidate, they must be the unique winner.

Alternatively: a method is Condorcet if any candidate who wins each head-to-head matchup must be the only winner.

(d) (3 points) Show that Coombs's method is not Condorcet.

Solution: This requires an example. Many examples are valid but one is

2	2	1
A	B	A
B	C	C
C	A	B

Here A beats B 3 to 2, and beats C 3 to 2, so A is Condorcet. But in Coombs's method, in the first round A and C tie for the most last-place votes and are eliminated simultaneously, so B is the unique winner and thus A does not win.

Another approach is this:

3	2	2	2
A	A	B	C
B	C	C	B
C	B	A	A

Here A beats either B or C five to four. But in Coombs's method, A has four last-place votes against C's three and B's two, so A is eliminated in the first round. (Then in the second round B has four last-place votes and C has five, so C is eliminated and B is the unique winner.)

Problem 2. (a) (4 points) Suppose we have an independent voting method, which may or may not be any of the methods we've studied in class. Suppose candidate A wins and candidate B loses in the first profile below. What can you say about the result in the second profile, and why?

3	2	2
B	A	C
C	C	B
A	B	A

$\rightarrow$

3	2	2
C	A	C
B	C	B
A	B	A

Solution: Candidate B cannot win in the second profile, because the relative rankings between A and B have not changed. Since the method is independent, and A wins and B loses in the first profile, B cannot win in the second.

(b) (6 points) Suppose we have an unknown voting method, which may or may not be any of the methods we've studied in class. In the first profile below, A is the unique winner, and in the second profile, B is the unique winner. Name two criteria this method does not satisfy, and explain why.

4	2	2
A	B	C
C	C	B
B	A	A

$\rightarrow$

4	2	2
A	B	C
C	C	A
B	A	B

Solution: This method isn't monotone, because A moves up in some rankings and this causes them to lose instead of win.

This method isn't anti-Condorcet, because in the second profile B is the anti-Condorcet candidate and still wins.

(We don't know if this method is Condorcet, because there's no Condorcet candidate in either profile. We don't know if it's Pareto, because no candidate is 100% preferred

to another. And we don't know if it's independent because the relative rankings of A and B *do* switch between the two profiles.)