Problem 1. (a) (5 points) For the following census, find the standard quota, lower quota, and Hamilton apportionment for $h = 10$.

k	p_k	q_k	$\lfloor q_k \rfloor$	Hamilton a_k
1	5,700			
2	3,100			
3	1,200			

Solution:

k	p_k	Standard Quota	Lower Quota	Hamilton Apportionment
1	5,700	5.7	5	6
2	3,100	3.1	3	3
3	1,200	1.2	1	1

(b) On the left, we have Census 1 and a set of apportionments for $h = 20$. (We don't know what apportionment method is being used, and it may not be a method we've discussed in class.)

Suppose we add a fourth state, to get Census 2 on the right. Give an apportionment for $h = 25$ that demonstrates the New States Paradox, and another that does not demonstrate the paradox. (You don't need to follow a specific apportionment method.)

Census 1		
k	p_k	$h = 20$ apportionment
1	5,000	9
2	3,000	4
3	4,500	7

Census 2			
k	p_k	$h = 25$ paradox	no paradox
1	5,000		
2	3,000		
3	4,500		
4	4,000		

Solution: There are lots of solutions here; here are a few examples.

Census 2			
k	p_k	$h = 25$ paradox	no paradox
1	5,000	8	10
2	3,000	4	4
3	4,500	8	7
4	4,000	5	4

Census 2			
k	p_k	$h = 25$ paradox	no paradox
1	5,000	10	9
2	3,000	3	4
3	4,500	7	7
4	4,000	5	5

Census 2			
k	p_k	$h = 25$ paradox	no paradox
1	5,000	8	9
2	3,000	5	4
3	4,500	6	6
4	4,000	6	6

For the paradox column, we need the numbers to add up to 25, but one of the first three numbers should be lower than in Census 1 and another should be higher. (The fourth number doesn't matter.)

For the no paradox column, we need the numbers to add up to 25, and *either* none of the first three states have gone down, *or* none of them have gone up.

Problem 2. (a) (2 points) What does it mean if we write that $p'_k < p_k$?

Solution: State k has a lower population in the second census than in the first census.

Alternatively: state k has lost population.

(b) For the following census, find the standard quota, lower quota for each state. Compute the next Jefferson critical divisor for each state, and use them to find the Jefferson apportionment for $h = 10$. What modified divisor did you pick? Show your modified quotas.

					$d =$	
k	p_k	q_k	$\lfloor q_k \rfloor$	Next CD	Modified Quota	Jefferson a_k
1	3,100					
2	2,500					
3	2,300					
4	2,100					

Solution: The most obvious value for d is 800, which I've worked out here. But anything between 775 and 833 will work.

					$d = 800$	
k	p_k	q_k	$\lfloor q_k \rfloor$	Next CD	Modified Quota	Jefferson a_k
1	3,100	3.1	3	775	3.875	3
2	2,500	2.5	2	833	3.125	3
3	2,300	2.3	2	767	2.875	2
4	2,100	2.1	2	700	2.625	2