Problem 1. Consider a census in which $p_1 = 6,000$, $p_2 = 17,000$, and $p_3 = 27,000$.

(a) Find the Adams apportionment for $h = 10$. Explicitly give the largest and smallest divisors that will lead to that apportionment.

(b) Find the Webster apportionment for $h = 10$. Explicitly give the largest and smallest divisors that will lead to that apportionment.

Solution:

(a) We need to compute critical divisors. The standard divisor is $s = 5000$, so the standard quotas are 1.2, 3.4, 5.4, with upper quotas 2,4,6, a total of twelve seats. We need to get rid of two.

The next standard divisors up are $6000/1 = 6000$, $17000/3 = 5667$, and $28000/5 = 5400$.

The next next standard divisors are $6000/0“ = ”\infty$, $17000/2 = 8500$, and $27000/4 = 6750$. So to get to ten seats, we need to be at least 5667 but below 6000.

This gives an apportionment of 2, 3, 5. We can see that either by counting critical divisors, or computing with something like $d = 5800$, which gives modified quotas of $1.03, 2.93, 4.66$.

(b) We can either compute critical divisors, or do trial and error.

We see the standard divisor is too big; all three standard quotas round down to 1, 3, 5.

For critical divisors, we compute $6000/1.5 = 4000$, $17000/3.5 = 4857.14$, and $27000/5.5 = 4909.09$. So we can pick any divisor between 4858 and 4909 inclusive.

For example, if we take $d = 4900$, we get modified quotas of $1.22, 3.47, 5.51$, which round to 1, 3, 6.

Problem 2. Consider a census in which $p_1 = 14,000$, $p_2 = 31,000$, and $p_3 = 55,000$.

(a) Find the Hill apportionment for $h = 10$.

(b) Find the Dean apportionment for $h = 10$.

Solution:

(a) The standard divisor works here. If $s = 10000$ then the standard quotas are $1.4, 3.1, 5.5$. For Hill's method, $1.4 < 1.414$ rounds down, and $3.1 < 3.464$ rounds down, but $5.5 > 5.477$ rounds up, so we get an apportionment of $1, 3, 6$.

(b) The standard divisor doesn't work here. For Dean's method, $1.4 > 1.333$ rounds up, $3.1 < 3.429$ rounds down, and $5.5 > 5.455$ rounds up, so that would apportion eleven seats.

But it's close. If we take $d = 10300$, say, then we get modified quotas of $1.359 > 1.333$, $3.010 < 3.429$, and $5.340 < 5.455$, which gives an apportionment of 2,3,5.

(Special mention for $d = 10500$, which gives standard quotas of 1.333, 2.952, and 5.238. It's actually not totally clear whether 1.333 should round up or down since that's *equal* to the Dean threshold. We didn't talk about that possibility in class. But I'll go ahead and accept it.)