Problem 1 (10 points)**.** Consider the game given by the following matrix:

-2	1	-3
-1	3	-1
-3	-2	4

(a) What is Row's greedy strategy?

(b) What is Column's greedy strategy?

(c) What is the payoff to the doubly greedy outcome?

(d) What is Row's prudent strategy, and why?

(e) What is Column's prudent strategy, and why?

(f) What is the doubly prudent outcome?

(g) Is there a saddle point? Why or why not?

Solution:

(a) Row 3 is greedy

(b) Column 1 or Column 3 is greedy

(c) Row 3, column 3, gives the payoff 4. Row 3, column 1 gives -3.

(d) We get the following guarantees:

-2	1	-3	-3
-1	3	-1	-1
-3	-2	4	-3
-1	3	4	

So Row's best guarantee is -1, which happens in row 2

(e) Column's best guarantee is -1, which happens in column 1.

(f) The doubly prudent outcome is row 2, column 1, which is -1.

(g) There is a saddle point at $(2,1)$ because $r = -1 = c$.

Problem 2 (6 points)**.** Consider the game given by the following matrix:

4	0	-3
-4	-1	-2
3	1	7
2	3	-2

(a) Fully reduce this game.

(b) Draw a flow diagram for the fully reduced game.

(c) Does this game have a saddle point? Why or why not?

Solution:

(a) We can remove row 2, since it's dominated by row 3. Then we can remove column 1, since it's now dominated by column 2. Then we can remove Row 1, since it's dominated by row 2.

Then we can remove row 2, since it's now dominated by row 1. Then we can remove column 4, since after that removal is is dominated by column 2.

4	0	-3
-4	-1	-2
3	1	7
2	3	-2

$\rightarrow$

4	0	-3
3	1	7
2	3	-2

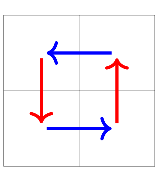
$\rightarrow$

0	-3
1	7
3	-2

$\rightarrow$

1	7
3	-2

(b) The flow diagram is



(c) In the flow diagram we can see there is no saddle point since no cell has all arrows pointing inward.

Problem 3 (4 points)**.** Suppose there is a lottery with a 1/10 chance of paying \$120 and a 4/10 chance of paying \$10.

(a) Describe the sample space.

(b) Explicitly write a probability distribution for this sample space.

(c) Compute the expected value of this lottery.

Solution:

(a) The sample space has three outcomes: win \$120, win \$10, or lose.

(b) $P = (1/10, 4/10, 5/10)$. (Or $P = (1/10, 2/5, 1/2)$.)

(c) $E = \frac{1}{10} \cdot 120 + \frac{4}{10} \cdot 10 + \frac{5}{10} \cdot 0 = 12 + 4 + 0 = 16$.