Math 1232 Spring 2026
Single-Variable Calculus 2
Mastery Quiz 10
Due Thursday, April 2

This week's mastery quiz has two topics. Everyone should submit work on both M3 and S8.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it.

Topics on This Quiz

- Major Topic 3: Series Convergence
- Secondary Topic 8: Power Series

Name:

Recitation Section:

M3: Series Convergence

(a) Analyze the convergence of the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n^3+n-1}{n^5-3n^4}$

Solution: We first want to check absolute convergence, so we first think about $\sum_{n=1}^{\infty} \frac{n^3+n-1}{n^5-3n^4}$.

Here it would be hard to use the regular comparison test. It's true that $\frac{n^3+n-1}{n^5-3n^4} \geq \frac{1}{n^2}$, but since this says it's greater than a convergent series, it doesn't really help.

Instead, we limit compare to $\frac{1}{n^2}$. We have

$$\lim_{n\to\infty} \frac{n^3+n-1/n^5-3n^4}{1/n^2} = \lim_{n\to\infty} \frac{n^5+n^3-n^2}{n^5-3n^4} = 1.$$

Since this is a finite non-zero number, the two series have the same convergence behavior. Thus, since $\frac{1}{n^2}$ converges, we know that $\sum_{n=1}^{\infty} \frac{n^3+n-1}{n^5-3n^4}$ also converges. Thus our original series converges absolutely.

(b) Analyze the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^3+n}$

Solution: We use the Ratio test. We have

$$\begin{aligned}\lim_{n\to\infty} \left| \frac{(-2)^{n+1}/(n+1)^3+n+1}{(-2)^n/n^3+n} \right| &= \lim_{n\to\infty} \frac{2(n^3+n)}{(n+1)^3+n+1} \\ &= \lim_{n\to\infty} 2 > 1.\end{aligned}$$

This limit is greater than 1, so by the ratio test this diverges.

Alternatively, we could note that

$$\lim_{n\to\infty} \frac{(-2)^n}{n^3+n} = \pm\infty,$$

so by the divergence test this diverges. But it's a little tricky to cleanly argue that this goes to infinity; we can't really use L'Hospital's rule without getting the negative sign out of there somehow.

(c) Analyze the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n2^n+1}$

Solution: You might try the ratio test here, but it won't actually help:

$$\lim_{n\to\infty}\left|\frac{{}^{(-2)^{n+1}}/_{(n+1)2^{n+1}+1}}{{}^{(-2)^n}/_{n2^n+1}}\right| = \lim_{n\to\infty}\frac{2(n2^n+1)}{(n+1)2^{n+1}+1} = \lim_{n\to\infty}\frac{n+1/2^{n+1}}{n+1+1/2^{n+1}} = 1.$$

Instead, we observe that this is an alternating series with the terms tending to zero, since

$$\lim_{n\to\infty}\frac{(-2)^n}{n2^n+1} = \lim_{n\to\infty}\frac{(-1)^n}{n+1/2^n} = 0.$$

Thus it converges. However, if we look at the absolute value, we can compare it to the series $\sum \frac{1}{n}$:

$$\lim_{n\to\infty}\frac{{}^{2^n}/_{n2^n+1}}{1/n} = \lim_{n\to\infty}\frac{n2^n}{n2^n+1} = 1$$

and since $\sum \frac{1}{n}$ diverges, by the limit comparison test our absolute-value series also diverges. Thus the original series converges conditionally.

S8: Power Series

(a) Find the radius of convergence and the interval of convergence of $\displaystyle\sum_{n=0}^{\infty}\frac{(n!)^2}{(3n)!}(x+2)^n$.

Solution: We use the ratio test.

$$\lim_{n\to\infty}\left|\frac{((n+1)!)^2(x+2)^{n+1}/(3n+3)!}{(n!)^2(x+2)^n/(3n)!}\right| = \lim_{n\to\infty}|x+2|\frac{(n+1)^2}{(3n+3)(3n+2)(3n+1)} \leq \frac{|x+2|}{n} = 0$$

for any x. So the radius of convergence is infinity, and this converges for all x.

(b) Find the radius of convergence and the interval of convergence of $\displaystyle\sum_{n=1}^{\infty}\frac{(2x-5)^n}{n^2}$.

Solution: We use the ratio test.

$$\begin{aligned}\lim_{n\to\infty}\left|\frac{{}^{(2x-5)^{n+1}}/_{(n+1)^2}}{{}^{(2x-5)^n}/_{n^2}}\right| &= \lim_{n\to\infty}\left|\frac{(2x-5)n^2}{(n+1)^2}\right| \\ &= |2x-5|\lim_{n\to\infty}\frac{n^2}{(n+1)^2} = |2x-5|.\end{aligned}$$

So we need $|2x-5| < 1$ or $-1 < 2x-5 < 1$, or $4 < 2x < 6$ or $2 < x < 3$. So the radius is $1/2$.

To find the interval we need to check the endpoints. We see

$$\sum_{n=0}^{\infty} \frac{(4-5)^n}{n^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2}$$

converges by alternating series test

$$\sum_{n=0}^{\infty} \frac{(6-5)^n}{n^2} = \sum_{n=0}^{\infty} \frac{1}{n^2}$$

converges by p-series test