

Math 1232: Single-Variable Calculus 2
George Washington University Spring 2026
Recitation 10

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Problem 1 (Infinite Decimals). We want to find a rational representation of the infinite decimal $0.\overline{47}$. That is, we want to write $0.\overline{47} = \frac{p}{q}$ for integers p, q .

- (a) First, what happens if we multiply $0.\overline{47}$ by 100?
- (b) Using part (a), what can you tell about $(99) \cdot 0.\overline{47}$?
- (c) Give a rational representation of $0.\overline{47}$.
- (d) Now let's take a different approach. Write $0.\overline{47}$ as an infinite series.
- (e) What kind of series is this? Can you use that fact to find a rational representation of $0.\overline{47}$?
- (f) Now use the same logic to find a rational representation of $2.\overline{63}$.

Problem 2. Consider the series $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^4}$.

- (a) Can you find a simpler series that's larger than this? One that's smaller?
- (b) Do the series you found in part (a) converge or diverge? Why? What does that tell you about our original series?
- (c) Now consider the series $\sum_{n=1}^{\infty} \frac{n^2 + 1}{n^4}$. Can you find a simpler series that's larger than this? One that's smaller?

- (d) Do the series you found in part (c) converge or diverge? What does that tell you about the convergence of the original series?
- (e) Now let's not worry about bigger or smaller. What is the simpler series that part (c) *kinda looks like*?
- (f) How can we use that to figure out if the series in part (c) converges or diverges?

Problem 3. For each of the following series, write a careful argument showing either that it converges or that it diverges. Think about exactly what test you want to use and why.

(a)
$$\sum_{n=2}^{\infty} \frac{5n^3 - 2}{3n^5 - n}$$

(b)
$$\sum_{n=2}^{\infty} \frac{n^3 \ln(n) + 1}{n^4 - 7}$$

(c)
$$\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$$

Problem 4 (Bonus). Does the series $\sum_{n=1}^{\infty} \frac{\sin^2(n^2 + e^n)}{n^2}$ converge or diverge?