Math 1232: Single-Variable Calculus 2
George Washington University   Spring 2026
Recitation 12

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Problem 1. Consider the function $f(x) = \dfrac{1}{1+x^6}$.

(a) Could you compute $\displaystyle\int \frac{1}{1+x^6}\,dx$? How?

(b) Does it help if I tell you that $1 + x^6 = (1+x^2)(x^2 - \sqrt{3}x + 1)(x^2 + \sqrt{3}x + 1)$?

(c) Now write a power series for $f(x)$ centered at 0. What is the interval of convergence?

(d) Compute the integral of your power series. What is the interval of convergence there?

Problem 2. We want to compute $\int_3^4 \frac{1}{1-(x-4)^3}\,dx$

(a) Find a power series for to compute $\frac{1}{1-(x-4)^3}$. Where is it centered? Will it converge from 3 to 4?

(b) Find an antiderivative for this power series. Where will it converge?

(c) Now compute the integral from 3 to 4. You should get a series. Does it converge? How could we predict that?

(d) Sum the first five terms with a calculator to estimate $\int_3^4 \frac{1}{1-(x-4)^3}\,dx$.

(e) Use an online integral calculator to find the integral. How close is your answer to the true answer?

(f) What would you predict about $\int_3^5 \frac{1}{1-(x-4)^3}\,dx$?

Problem 3. Let's find the Taylor series of $f(x) = e^x$ centered at $a = 1$.

(a) Compute f', f'', f'''. Find a formula for $f^{(n)}(x)$.

(b) Give a formula for $T_f(x, 1)$.

(c) We want to know if $f(x) = T_f(x, 1)$. Find a formula for $R_k(x, 1)$. Can you show this goes to 0 as k goes to infinity?

(d) We already have another power series for f:

$$T_f(x, 0) = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

You should have a different power series; but can you convince yourself it *should* give the same function? (What happens if you plug $x - 1$ into this series?)

Problem 4. Let's do something silly, and compute the Taylor series of a polynomial.

(a) Let $f(x) = x^3 + 3x^2 + 1$. Find the Taylor series centered at zero. Was that what you expected?

(b) Now find the Taylor series centered at 2. Do you get the same thing? What's useful about this?

Problem 5. Back in section 2 we talked about the bell curve function $p(x) = e^{-x^2}$. (Technically we should be talking about $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ but that's annoying and doesn't change the details enough to be interesting.)

(a) Find a power series for $p(x)$ centered at zero. (This should not require any real calculations.)

(b) Find an antiderivative for $p(x)$, using power series.

(c) Write down a series that computes $\int_0^1 p(x)\, dx$.

(d) Add up the first three or four terms of this series. What do you get? Can you estimate the error in this calculation?