Math 1232 Spring 2026
Single-Variable Calculus 2
Mastery Quiz 2
Due Thursday, January 29

This week's mastery quiz has two topics. Everyone should submit work on topic M1: Calculus of Transcendental Functions. If you have a 2/2 on topic S1: Invertible Functions, you don't need to submit it again. If you have a 1/2 or a 0/2, you should submit answers to this week's questions on S1. Check Blackboard if you're uncertain of your score.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it.

Topics on This Quiz

- Major Topic 1: Calculus of Transcendental Functions
- Secondary Topic 1: Invertible Functions

Name:

Recitation Section:

M1: Calculus of Transcendental Functions

(a) Compute $\displaystyle\int \frac{\sin(2t)+\cos(2t)}{\sin(2t)-\cos(2t)}\,dt$.

Solution: We can take $u = \sin(2t) - \cos(2t)$. Then $du = (2\cos(2t) + 2\sin(2t))dt$, and we get

$$\begin{aligned}\int \frac{\sin(2t)+\cos(2t)}{\sin(2t)-\cos(2t)}\,dt &= \int \frac{\sin(2t)+\cos(2t)}{u}\frac{du}{2\cos(2t)+2\sin(2t)}\\ &= \int \frac{1}{2}u\,du = \frac{1}{2}\ln|u| + C\\ &= \frac{1}{2}\ln|\sin(2t)-\cos(2t)| + C.\end{aligned}$$

(b) Compute $\displaystyle\frac{d}{dx}x^{\ln(x)}$.

Solution: The simplest approach is to use logarithmic differentiation.

$$\begin{aligned}y &= x^{\ln(x)}\\ \ln(y) &= \ln(x)\ln(x) = \ln(x)^2\\ \frac{y'}{y} &= 2\ln(x)\frac{1}{x}\\ y &= \frac{2\ln(x)}{x}y = \frac{2\ln(x)x^{\ln(x)}}{x}.\end{aligned}$$

Alternatively, we could compute

$$\begin{aligned}\frac{d}{dx}x^{\ln(x)} &= \frac{d}{dx}\left(e^{\ln(x)}\right)^{\ln(x)} = \frac{d}{dx}e^{\ln(x)^2}\\ &= e^{\ln(x)^2}\cdot 2\ln(x)\frac{1}{x} = \frac{2\ln(x)x^{\ln(x)}}{x}.\end{aligned}$$

(c) Compute $\displaystyle\int \frac{3^x}{\sqrt{1-3^{2x}}}\,dx$.

Solution: Set $u = 3^x$, so that $du = 3^x\ln(3)\,dx$. Then

$$\begin{aligned}\int \frac{3^x}{\sqrt{1-3^{2x}}}\,dx &= \int \frac{1}{\ln(3)}\frac{1}{\sqrt{1-u^2}}\,du\\ &= \frac{1}{\ln(3)}\arcsin(u) + C = \frac{1}{\ln(3)}\arcsin(3^x) + C.\end{aligned}$$

S1: Invertible Functions

(a) Showing your work, compute $\log_3(90) + \log_3(3/2) - \log_3(5)$. Your answer should be an integer.

Solution:

$$\log_3(90) + \log_3(3/2) - \log_3(5) = \log_3(90 \cdot 3/2/5) = \log_3(27) = 3.$$

(b) Compute $\tan(\arcsin(2/5))$.

Solution: If we draw a triangle, we have opposite side with length 2 and hypotenuse with length 5. By the Pythagorean theorem, the adjacent side will have length $\sqrt{21}$, and thus $\tan(\theta) = \frac{2}{\sqrt{21}}$.

(c) Let $h(x) = x^5 + x$. Compute $(h^{-1})'(2)$.

Solution: By the Inverse Function Theorem, we know that

$$(h^{-1})'(2) = \frac{1}{h'(h^{-1}(2))}.$$

Guess and check shows that $h(1) = 2$ so $h^{-1}(2) = 1$. And we know that

$$h'(x) = 5x^4 + 1$$

and thus

$$h'(1) = 5 + 1 = 6$$

Thus

$$(h^{-1})'(2) = \frac{1}{6}.$$

(d) Is $f(x) = \sqrt{x^4 + 1}$ invertible? If so, find a formula for the inverse. If not, explain why not. Justify your answer.

Solution: We have $f(-1) = f(1) = \sqrt{2}$ so this function is not one-to-one, and thus not invertible.