

Math 1232 Spring 2026
Single-Variable Calculus 2
Mastery Quiz 3
Due Thursday, February 5

This week's mastery quiz has two topics. Everyone should submit work on both topics. Even if you got a 2 on M1 last week, you should submit it again, since you need to show mastery twice on major topics.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it.

Topics on This Quiz

- Major Topic 1: Calculus of Transcendental Functions
- Secondary Topic 2: L'Hospital's Rule

Name:

Recitation Section:

M1: Calculus of Transcendental Functions

(a) Compute $\int \frac{x^2}{1+x^6} dx$

Solution: If we take $u = x^3$ so $du = 3x^2 dx$, we have

$$\begin{aligned} \int \frac{x^2}{1+x^6} dx &= \int \frac{1}{3} \frac{1}{1+u^2} du \\ &= \frac{1}{3} \arctan(u) + C = \frac{1}{3} \arctan(x^3) + C. \end{aligned}$$

(b) (Note this is a definite integral)

$$\int_0^2 \frac{e^x}{e^x+1} dx =$$

Solution: We can take $u = e^x$ so $du = e^x dx$ and

$$\int_0^2 \frac{e^x}{e^x+1} dx = \int_1^{e^2} \frac{1}{u+1} du = \ln|u+1| \Big|_1^{e^2} = \ln(e^2+1) - \ln(2).$$

(c) Compute $\frac{d}{dx} x^{\sqrt{x^2+1}}$

Solution:

$$\begin{aligned} y &= x^{\sqrt{x^2+1}} \\ \ln|y| &= \sqrt{x^2+1} \ln|x| \\ y'/y &= \frac{x \ln|x|}{\sqrt{x^2+1}} + \frac{\sqrt{x}}{x} \\ y' &= x^{\sqrt{x^2+1}} \left(\frac{x \ln|x|}{\sqrt{x^2+1}} + \frac{\sqrt{x}}{x} \right) \end{aligned}$$

S2: L'Hospital's Rule

(a) $\lim_{x \rightarrow \infty} \sqrt{x} e^{-x} =$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} \sqrt{x} e^{-x} &= \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{e^x} = \lim_{x \rightarrow \infty} \frac{1/2\sqrt{x}}{e^x} \\ &= \lim_{x \rightarrow \infty} \frac{1}{2e^x \sqrt{x}} = 0. \end{aligned}$$

(b) $\lim_{x \rightarrow \infty} x^{\ln(3)/(2+\ln(x))} =$

Solution:

$$\ln(y) = \frac{\ln(3) \ln(x)}{2 + \ln(x)}$$

$$\lim_{x \rightarrow \infty} \frac{\ln(3) \ln(x) \nearrow \infty}{2 + \ln(x) \searrow \infty} = \text{L'H} \lim_{x \rightarrow \infty} \frac{\ln(3)/x}{1/x} = \ln(3)$$

$$\lim_{x \rightarrow \infty} y = e^{\ln(3)} = 3.$$

(c) $\lim_{x \rightarrow 0} \frac{x^3 - x^2}{x + \sin(x)} =$

Solution: The limits of the top and bottom are both zero, so we can use L'Hospital's Rule:

$$\lim_{x \rightarrow 0} \frac{x^3 - x^2 \nearrow 0}{x + \sin(x) \searrow 0} = \text{L'H} \lim_{x \rightarrow 0} \frac{3x^2 - 2x \nearrow 0}{1 + \cos(x) \searrow 2} = \frac{0}{2} = 0.$$

Note we *cannot* use L'Hospital's rule a second time, because we don't have an indeterminate form.