

Math 1232: Single-Variable Calculus 2  
George Washington University    Spring 2026  
Recitation 3

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January 28, 2026

**Problem 1.** Consider the integral  $\int_e^{e^4} \frac{1}{x\sqrt{\ln x}} dx$ .

- (a) We're going to have to do a  $u$ -substitution here. What  $u$  looks like it should work?
- (b) What do we need to change the bounds to when we do the  $u$ -substitution?
- (c) Compute  $\int_e^{e^4} \frac{1}{x\sqrt{\ln x}} dx$ .
- (d) Now try computing  $\int \frac{1}{x\sqrt{\ln x}} dx$  to get the antiderivative.
- (e) Now plug  $e^4$  and  $e$  in to your antiderivative. What do you notice? How is this related to part (c)?

**Solution:**

- (a) We take  $u = \ln(x)$ , and  $du = \frac{dx}{x}$ . This seems plausible because  $\ln(x)$  is on the inside of a function.
- (b)  $\ln(e) = 1$  and  $\ln(e^4) = 4$ , so we have to integrate from 1 to 4.

(c)

$$\int_e^{e^4} \frac{1}{x\sqrt{\ln x}} dx = \int_1^4 \frac{1}{\sqrt{u}} du = 2\sqrt{u} \Big|_1^4 = 4 - 2 = 2.$$

(d)

$$\int \frac{1}{x\sqrt{\ln x}} dx = \int \frac{1}{\sqrt{u}} du = 2\sqrt{u} + C = 2\sqrt{\ln(x)} + C.$$

- (e) We get  $2\sqrt{\ln(e^4)} = 2\sqrt{4} = 4$  and  $2\sqrt{\ln(e)} = 2\sqrt{1} = 2$ , which are the same numbers we got before. And specifically, you see we get 4 and 1 as intermediate answers here—these are the same numbers we got by doing the change of bounds.

**Problem 2.** Compute the following integrals.

(a)  $\int \frac{e^x}{1+e^x} dx.$

(b)  $\int \frac{\ln(x)}{x} dx.$

**Solution:**

- (a) Take  $u = 1 + e^x$  so  $dx = \frac{du}{e^x}$ . Then

$$\int \frac{e^x}{1+e^x} dx = \int \frac{e^x du}{u e^x} = \int \frac{du}{u} = \ln |u| + C = \ln |1 + e^x| + C.$$

- (b) This one looks tricky, and you might have to mess around with it a bit to see, and try different things. But if we take  $u = \ln(x)$  so that  $dx = x du$ , we see this is

$$\int u du = \frac{u^2}{2} + C = \frac{(\ln |x|)^2}{2} + C.$$

**Problem 3 (Challenge).** Compute  $\int \frac{dx}{1+e^x}$ .

**Solution:** This problem becomes much easier if we multiply the top and bottom by  $e^{-x}$ .

Then we have  $\int \frac{e^{-x}}{e^{-x} + 1} dx$ . Set  $u = e^{-x}$  so that  $du = -e^{-x} dx$  and we have

$$\int \frac{e^{-x}}{e^{-x} + 1} dx = - \int \frac{du}{1+u} = -\ln(1+u) = -\ln(1+e^{-x}).$$

Alternatively, we can take  $u = e^x$ ,  $du = e^x dx$ , and have

$$\int \frac{dx}{1+e^x} = \int \frac{du}{u(u+1)}.$$

Again nonobviously, we write

$$\begin{aligned} \int \frac{du}{u(u+1)} &= \int \frac{1+u-u}{u(u+1)} du = \int \frac{1+u}{u(u+1)} du - \int \frac{u}{u(u+1)} du \\ &= \int \frac{du}{u} - \int \frac{du}{u+1} \\ &= \ln(u) - \ln(u+1) = \ln(e^x) - \ln(e^x + 1). \end{aligned}$$

Using properties of logs, you can check that this is the same as the previous answer. Or, if you prefer, you can write  $x - \ln(e^x + 1)$ .

**Problem 4.** (a) Compute  $\sin(\arctan(5))$ .

(b) Compute  $\frac{d}{dx} \arccos(\sqrt{x})$

(c) Compute  $\frac{d}{dx} \arctan(x + \sec(x))$

**Solution:**

(a) Our implicit triangle has side lengths of 5, 1,  $\sqrt{26}$ . So  $\sin(\arctan(5)) = \frac{5}{\sqrt{26}}$ .

(b)

$$\frac{d}{dx} \arccos(\sqrt{x}) = \frac{-1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}.$$

(c)

$$\frac{d}{dx} \arctan(x + \sec(x)) = \frac{1}{1 + (x + \sec(x))^2} \cdot (1 + \sec(x) \tan(x)).$$

**Problem 5.** Compute the following integrals:

(a)  $\int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx.$

(b)  $\int_0^1 \frac{e^{2x}}{1+e^{4x}} dx.$

**Solution:**

(a) Take  $u = \arcsin(x)$ , and  $du = \frac{dx}{\sqrt{1-x^2}}$ . Then

$$\int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx = \int u du = \frac{u^2}{2} + C = \frac{1}{2}(\arcsin(x))^2 + C.$$

(b) Set  $u = e^{2x}$  so  $du = 2e^{2x} dx$ .  $g(0) = 1$  and  $g(1) = e^2$ . Then

$$\int_0^1 \frac{e^{2x}}{1+e^{4x}} dx = \int_1^{e^2} \frac{1}{2(1+u^2)} du = \frac{1}{2} \arctan(u) \Big|_1^{e^2} = \frac{1}{2} (\arctan(e^2) - \arctan(1)).$$

**Problem 6** (Completing the Square). Sometimes to do an inverse trig integral we need to *complete the square*. This is something you probably learned in high school then immediately forgot.

Let's say we want to compute  $\int \frac{dx}{x^2-2x+2}$ .

(a) Is  $x^2 - 2x + 2$  a perfect square?

- (b) We want to write  $x^2 - 2x + 2$  in the form  $u^2 + c$ , where  $c$  is a constant. That means we need to find some other number  $a$  such that  $x^2 - 2x + a$  is a perfect square. What value of  $a$  will work here?
- (c) Rewrite  $x^2 - 2x + 2 = x^2 - 2x + a + b$ , where  $a$  is the number you just found, and  $b$  is whatever's left over.
- (d) Set  $u^2 = x^2 - 2x + a$ , and complete your  $u$ -substitution.
- (e) Use that substitution and the derivative of arctan to compute  $\int \frac{dx}{x^2 - 2x + 2}$ .

**Solution:**

- (a) Definitely not.
- (b) All perfect squares have the form  $R^2 + 2RS + S^2$ . In this case we must have  $R = x$ , and thus  $2RS = 2xS = -2x$ , so  $S = -1$ . Then  $S^2 = 1$ . We take  $a = 1$ .
- (c)  $x^2 - 2x + 2 = x^2 - 2x + 1 + 1 = (x - 1)^2 + 1$ .
- (d) Take  $u = x - 1$ , so that  $du = dx$ .
- (e)

$$\begin{aligned} \int \frac{dx}{x^2 - 2x + 2} &= \int \frac{dx}{(x - 1)^2 + 1} \\ &= \int \frac{du}{u^2 + 1} \\ &= \arctan(u) + C = \arctan(x - 1) + C. \end{aligned}$$