Math 1232 Spring 2026
Single-Variable Calculus 2
Mastery Quiz 4
Due Thursday, February 12

This week's mastery quiz has three topics. Everyone should submit work on M2. If you have a 4/4 on M1, or a 2/2 on S2, you don't need to submit them again. (Please check Blackboard to confirm your scores!)

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it.

Topics on This Quiz

- Major Topic 1: Calculus of Transcendental Functions
- Major Topic 2: Advanced Integration Techniques
- Secondary Topic 2: L'Hospital's Rule

Name:

Recitation Section:

M1: Calculus of Transcendental Functions

(a) $\int \frac{1}{x\sqrt{4-\ln(x)^2}}\,dx =$

Solution: We can set $u = \ln(x)/2$ so $du = \frac{1}{2x}\,dx$. Then

$$\int \frac{1}{x\sqrt{4-\ln(x)^2}}\,dx = \int \frac{2}{\sqrt{4-4u^2}}\,du = \int \frac{1}{\sqrt{1-u^2}}\,du$$
$$= \arcsin(u) + C = \arcsin(\ln(x)/2) + C.$$

(b) Compute $\frac{d}{dx}x^{e^x}$

Solution:

$$\begin{aligned} y &= x^{e^x} \\ ln|y| &= e^x \ln|x| \\ y'/y &= e^x \ln|x| + \frac{e^x}{x} \\ y' &= e^x \ln|x| x^{e^x} + \frac{1}{x}e^x x^{e^x}. \end{aligned}$$

(c) $\int \sec^2(x)e^{\tan(x)}\,dx =$

Solution: We take $u = \tan(x)$ so $du = \sec^2(x)\,dx$. Then

$$\int \sec^2(x)e^{\tan(x)}\,dx = \int e^u\,du \qquad = e^u + C = e^{\tan(x)} + C.$$

M2: Advanced Integration Techniques

(a) $\int e^x \sin(5x)\,dx =$

Solution:

$$\int e^x \sin(5x)\,dx = e^x \sin(5x) - \int 5e^x \cos(5x)\,dx$$
$$\int e^x \cos(5x) = e^x \cos(5x) + \int 5e^x \sin(5x)\,dx$$
$$\int e^x \sin(5x)\,dx = e^x \sin(5x) - 5e^x \cos(5x) - 25\int e^x \sin(5x)\,dx$$
$$26\int e^x \sin(5x)\,dx = e^x \sin(5x) - 5e^x \cos(5x) + C$$
$$\int e^x \sin(5x)\,dx = \frac{1}{26}e^x \sin(5x) - \frac{5}{26}e^x \cos(5x) + C.$$

(b) $\displaystyle\int \tan^4(3x)\,dx.$

Solution:

$$\begin{aligned}\int \tan^4(3x)\,dx &= \int (\sec^2(3x)-1)\tan^2(3x)\,dx = \int \sec^2(3x)\tan^2(3x) - \tan^2(3x)\,dx\\ &= \int \sec^2(3x)\tan^2(3x) - \big(\sec^2(3x)-1\big)\,dx = \int \sec^2(3x)\tan^2(3x) - \sec^2(3x) + 1\,dx\\ &= \frac{1}{9}\tan^3(3x) - \frac{1}{3}\tan(3x) + x + C.\\ &\left(= \frac{1}{9}\sec^2(3x)\tan(3x) - \frac{4}{9}\tan(3x) + x + C.\right)\end{aligned}$$

(c) $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}} =$

Solution: We can take $x = 2\sin(\theta)$, so that $dx = 2\cos(\theta)\,d\theta$. Then we get

$$\begin{aligned}\int \frac{dx}{x^2\sqrt{4-x^2}} &= \int \frac{2\cos(\theta)\,d\theta}{4\sin^2(\theta)\sqrt{4-4\sin^2(\theta)}}\\ &= \int \frac{2\cos(\theta)\,d\theta}{4\sin^2(\theta)\sqrt{4\cos^2(\theta)}}\\ &= \int \frac{2\cos(\theta)\,d\theta}{4\sin^2(\theta)\cdot 2\cos(\theta)}\\ &= \int \frac{d\theta}{4\sin^2(\theta)} = \int \frac{1}{4}\csc^2(\theta)\,d\theta\\ &= -\frac{1}{4}\cot(\theta) + C.\end{aligned}$$

Now we need to substitute our x back in. We know that $\sin(\theta) = x/2$, so we can construct a right triangle with opposite side x, hypotenuse 2, and thus adjacent side $\sqrt{4-x^2}$. (Note that this is the term that showed up in the original integral!)

Then $\cot(\theta) = \frac{\text{adjacent}}{\text{opposite}} = \frac{\sqrt{4-x^2}}{x}$, and so we have

$$\int \frac{dx}{x^2\sqrt{4-x^2}} = -\frac{1}{4}\cot(\theta) + C = -\frac{\sqrt{4-x^2}}{4x} + C.$$

S2: L'Hospital's Rule

(a) $\lim_{x\to 0}(2x+1)^{\cot(x)} =$

Solution: Taking logs of both sides gives

$$y = (2x+1)^{\cot(x)}$$
$$\ln|y| = \cot(x)\ln|2x+1| = \frac{\cos(x)\ln|2x+1|}{\sin(x)}.$$

The top and bottom both approach 0, so we can use L'Hospital's Rule:

$$\lim_{x\to 0}\ln|y| = \lim_{x\to 0}\cos(x)\frac{\ln|2x+1|}{\sin(x)} = 1\cdot\lim_{x\to 0}\frac{\ln|2x+1|^{\nearrow 0}}{\sin(x)_{\searrow 0}}$$
$$=^{\text{L'H}}\lim_{x\to 0}\frac{2/(2x+1)}{\cos(x)} = 2$$
$$\lim_{x\to 0} y = e^2.$$

(b) $\lim_{x\to 0}\frac{\arctan(x)}{\ln(x+1)} =$

Solution:

$$\lim_{x\to 0}\frac{\arctan(x)^{\nearrow 0}}{\ln(x+1)_{\searrow 0}} =^{\text{L'H}} \lim_{x\to 0}\frac{1/(x^2+1)}{1/(x+1)} = \lim_{x\to 0}\frac{x+1}{x^2+1} = 1.$$

(c) $\lim_{x\to+\infty}\frac{\arctan(x)}{\arctan(x)+1} =$

Solution: $\lim_{x\to+\infty} \arctan(x) = \pi/2$, so this limit is $\frac{\pi/2}{\pi/2+1} \approx .611$.

Note: you cannot use L'Hospital's rule here! If you tried, you would get

$$\lim_{x\to+\infty} \frac{1/(x^2+1)}{1/(x^2+1)} = \lim_{x\to+\infty} 1 = 1$$

but that is not in fact the limit.