Math 1232 Spring 2026
Single-Variable Calculus 2
Mastery Quiz 5
Due Thursday, February 19

This week's mastery quiz has four topics. Everyone should submit work on M2, S3, and S4. If you have a 4/4 on M1, you don't need to submit it again. (Please check Blackboard to confirm your scores!) This will be the last quiz featuring M1.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it.

Topics on This Quiz

- Major Topic 1: Calculus of Transcendental Functions

- Major Topic 2: Advanced Integration Techniques

- Secondary Topic 3: Numeric Integration

- Secondary Topic 4: Improper Integrals

Name:

Recitation Section:

M1: Calculus of Transcendental Functions

(a) Compute $\dfrac{d}{dx}\cos(x)^{\sin(x)}$

Solution:

$$y = \cos(x)^{\sin(x)}$$
$$ln|y| = \sin(x)\ln|\cos(x)|$$
$$y'/y = \cos(x)\ln|cos(x)| + \sin(x)\frac{-\sin(x)}{\cos(x)}$$
$$y' = \cos(x)^{-\sin(x)}\left(\cos(x)\ln|\cos(x)| - \frac{\sin^2(x)}{\cos(x)}\right)$$

(b) Compute $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\,dx$

Solution: We take $u = 1/x$ so $du = \frac{-1}{x^2}\,dx$. Then

$$\int_1^2 \frac{e^{1/x}}{x^2}\,dx = \int_1^{1/2} -e^u\,du$$
$$= -e^u\big|_1^{1/2} = -e^{1/2} + e^1 = e - \sqrt{e}.$$

(c) Compute $\displaystyle\int \frac{1}{x + x(\ln(x))^2}\,dx.$

Solution: Set $u = \ln(x)$ so $du = dx/x$. Then

$$\int \frac{1}{x + x\ln(x)^2}\,dx = \int \frac{1}{1+u^2}\,du = \arctan(u) + C = \arctan(\ln(x)) + C.$$

M2: Advanced Integration Techniques

(a) Compute $\int \frac{x^3 - x + 2}{x+1}\,dx =$

Solution: Polynomial long division gives that

$$x^3 - x + 2 = x^2(x+1) - x(x+1) + 2$$

$$\frac{x^3 - x + 2}{x + 1} = x^2 - x + \frac{2}{x+1}$$

$$\int \frac{x^3 - x + 2}{x + 1}\, dx = \int x^2 - x + \frac{2}{x+1}\, dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + 2\ln|x+1| + C.$$

(b) Compute $\displaystyle\int \frac{4x^2 - x + 10}{(x-2)(x^2+4)}\, dx$.

Solution: We need to do a partial fractions decomposition. We have

$$\frac{4x^2 - x + 10}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx + C}{x^2+4}$$

$$4x^2 - x + 10 = A(x^2+4) + (Bx+C)(x-2).2: \quad 24 \qquad\qquad\qquad = A \cdot 8$$

$$A = 30: \quad 10 \qquad\qquad\qquad\qquad = 4A + (C)(-2) = 12 - 2C$$

$$C = 6 - 5 = 1$$

$$1: \quad 13 = 3(1+4) + (B+1)(1-2) = 15 - B - 1$$

$$B = 1.$$

Thus we compute

$$\int \frac{4x^2 - x + 10}{(x-2)(x^2+4)}\, dx = \int \frac{3}{x-2} + \frac{x+1}{x^2+4}\, dx$$

$$= \int \frac{3}{x-2} + \frac{x}{x^2+4} + \frac{1}{x^2+4}\, dx$$

$$= 3\ln|x-2| + \frac{1}{2}\ln|x^2+4| + \frac{1}{4}\int \frac{1}{(x/2)^2+1}\, dx$$

$$= 3\ln|x-2| + \frac{1}{2}\ln|x^2+4| + \frac{1}{2}\arctan(x/2) + C.$$

(c) $\displaystyle\int \frac{\sqrt{4x^2 - 1}}{x}\, dx =$

Solution: We set $2x = \sec(\theta)$, so $dx = \frac{1}{2}\sec(\theta)\tan(\theta)\, d\theta$, and

$$\int \frac{\sqrt{4x^2 - 1}}{x}\, dx = \int \frac{\sqrt{\sec^2 \theta - 1}}{\frac{1}{2}\sec \theta}\frac{1}{2}\sec(\theta)\tan \theta\, d\theta$$

$$= \int \tan^2(\theta)\, d\theta = \int \sec^2(\theta) - 1\, d\theta$$

$$= \tan(\theta) - \theta + C$$

Then we know $\sec \theta = 2x$ so we can make a triangle with hypotenuse $2x$ and adjacent side 1, and thus opposite side $\sqrt{4x^2 - 1}$, so $\tan(\theta) = \sqrt{4x^2 - 1}$. Then we can say either $\theta = \operatorname{arcsec}(2x)$ or $\theta = \arctan(\sqrt{4x^2 - 1})$, and we have

$$\int \frac{\sqrt{4x^2 - 1}}{x}\, dx = \sqrt{4x^2 - 1} - \arctan(\sqrt{4x^2 - 1}) + C = \sqrt{4x^2 - 1} - \operatorname{arcsec}(2x) + C.$$

S3: Numeric Integration

(a) How many intervals do you need with the **trapezoid** rule to approximate $\int_2^5 \frac{x}{x+1}$ to within $1/200$? Compute that approximation. (Feel free to use a calculator to plug in numeric values, or to leave the answer in exact unsimplified terms, but show every step.)

Solution: We have

$$f'(x) = \frac{(x+1) - x}{(x+1)^2} = \frac{1}{(x+1)^2}$$

$$f''(x) = \frac{-2}{(x+1)^3}$$

$$|f''(x)| \le \frac{2}{3^3} = \frac{2}{27}.$$

Then we have

$$|E_T| \le \frac{2/27 \cdot 3^3}{12n^2} = \frac{1}{6n^2}.$$

We want $1/200 \ge \frac{1}{6n^2}$ which implies

$$n^2 \ge \frac{200}{6} = 100/3 \approx 33$$

$$n \ge 6.$$

Then we have

$$T_6 = \frac{1}{4}f(2) + \frac{1}{2}f(2.5) + \frac{1}{2}f(3) + \frac{1}{2}f(3.5) + \frac{1}{2}f(4) + \frac{1}{2}f(4.5) + \frac{1}{4}f(5)$$
$$= \frac{1}{6} + \frac{5}{14} + \frac{3}{8} + \frac{7}{18} + \frac{4}{10} + \frac{9}{22} + \frac{5}{24}$$
$$= \frac{31949}{13860} \approx 2.30512.$$

Since the true answer is $3 - \ln(2) \approx 2.30685$ this is in fact within our margin of error.

(b) Suppose we have

$$g(0) = 5 \quad g(1) = 4 \quad g(2) = 2 \quad g(3) = 3 \quad g(4) = 5 \quad g(5) = 6 \quad g(6) = 5$$

Approximate $\int_0^6 g(x)\,dx$ as accurately as you can using the midpoint rule, and again using Simpson's rule.

Solution: For the midpoint rule, we have

$$T_3 = 2g(1) + 2g(3) + 2g(5) = 8 + 6 + 12 = 26.$$

For the Simpson's rule, we have

$$S_6 = \frac{1}{3}(5 + 16 + 4 + 12 + 10 + 24 + 5) = \frac{76}{3} \approx 25.33.$$

S4: Improper Integrals

Solution:

(a) Compute $\displaystyle\int_1^\infty \frac{dx}{x^4} =$

   Solution:

$$\int_1^\infty \frac{dx}{x^4} = \lim_{t\to\infty} \int_1^t \frac{dx}{x^4}$$
$$= \lim_{t\to\infty} \frac{-1}{3x^3}\Big|_1^t$$
$$= \lim_{t\to\infty} \frac{1}{3} - \frac{1}{3t^3} = \frac{1}{3}.$$

(b) Compute $\displaystyle\int_0^7 \frac{1}{\sqrt[3]{7-x}}\,dx.$

Solution: We know that $\frac{1}{\sqrt[3]{7-x}}$ is undefined at 7. So we have

$$\int_0^7 \frac{1}{\sqrt[3]{7-x}}\,dx = \lim_{t \to \pi/2} \int_0^t \frac{1}{\sqrt[3]{7-x}}\,dx$$

$$= \lim_{t \to \pi/2} -\frac{3}{2}(7-x)^{2/3}$$

$$= \lim_{t \to \pi/2} -0 + \frac{3}{2}(7-t)^{2/3} = \frac{3}{2}7^{2/3}.$$