

Math 1232: Single-Variable Calculus 2
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Recitation 6

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Problem 1. We want to find the cross-sectional area of a two-meter-long airplane wing. We measure its width every 20 centimeters, and get: 5.8, 20.3, 26.7, 29.0, 27.6, 27.3, 23.8, 20.5, 15.1, 8.7, 2.8. Use the trapezoidal rule and Simpson's rule to estimate the area of the wing.

Problem 2. Consider the function $f(x) = x^2 + 1$.

- (a) Use the trapezoid rule with six intervals to estimate $\int_{-4}^2 f(x) dx$.
- (b) Use the midpoint rule with six intervals to estimate $\int_{-4}^2 f(x) dx$.
- (c) Use Simpson's rule with six intervals to estimate $\int_{-4}^2 f(x) dx$.
- (d) Which of these do you expect to be most accurate? Which do you expect to be least accurate?
- (e) Compute $\int_{-4}^2 f(x) dx$. What do you find? Why?

Problem 3. Let $g(x) = e^{-x^2}$, and suppose we want to compute $\int_{-1}^2 e^{-x^2} dx$, and get the answer correct to two decimal places.

- (a) We can compute that $g''(x)$ varies between -2 and $.9$ when x is in $[-1, 2]$. What value should we take for K ?
- (b) How many subintervals should we use to get the answer correct to within two decimal places using the trapezoid rule?

- (c) How many subintervals should we use to get the answer correct to within two decimal places using the midpoint rule?
- (d) We can compute that $g'''(x)$ varies between -8 and 12 . What value should we take for L ?
- (e) How many subintervals should we use to get the answer correct to within two decimal places using Simpson's rule?

Problem 4. We want to compute $\int_0^2 \frac{1}{\sqrt[3]{x-1}} dx$.

- (a) Can you compute an antiderivative? Can you evaluate it at 0 and 2?
- (b) Did part (a) finish the problem? Sketch a picture of the graph. What should we be concerned about?
- (c) Carefully set up a computation that will find $\int_0^2 \frac{1}{\sqrt[3]{x-1}} dx$. (Hint: It should have two limit operations in it.)
- (d) What did we learn from this that we didn't learn from (a)?

Problem 5 (Bonus). We want to figure out if $\int_0^{+\infty} e^{-x^2} dx$ converges—that is, if it's finite or infinite.

- (a) If we can find an antiderivative, we can just compute the improper integral directly. Why doesn't that work?
- (b) Since we can't integrate this directly we might want to use a comparison test. We need to find an easy-to-integrate function that's larger than e^{-x^2} . Find a function $f(x)$ that makes $f(x)e^{-x^2}$ easy to integrate.
- (c) If $f(x) \geq 1$, then we can just integrate $f(x)e^{-x^2}$. Is it?
- (d) This is where we can pull in a trick. Is there some a where $f(x) > 1$ when $x > a$? (You may need to adjust your $f(x)$ here, especially the sign. It's fine as long as you can still integrate it.)
- (e) We know $\int_a^{+\infty} e^{-x^2} dx \leq \int_a^{+\infty} f(x)e^{-x^2} dx$. Compute the new improper integral; is it finite?

(f) Now we just have to deal with $\int_0^a e^{-x^2} dx$. We can't do that integral exactly, but that's fine: you should be able to tell whether it's finite or not without doing any calculations. How?

(g) Does $\int_0^{+\infty} e^{-x^2} dx$ converge?