Math 1232 Spring 2026
Single-Variable Calculus 2
Mastery Quiz 7
Due Thursday, March 5

This week's mastery quiz has three topics. Everyone should submit work on S6, unless you're *really* confident you aced it on the midterm. If you have a 4/4 on M2, or a 2/2 on S5, you don't need to submit them again. (Please check Blackboard to confirm your scores!) This will be the last quiz featuring M2 or S5.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it.

Topics on This Quiz

- Major Topic 2: Advanced Integration Techniques
- Secondary Topic 5: Geometric Applications
- Secondary Topic 6: Differential Equations

Name:

Recitation Section:

M2: Advanced Integration Techniques

(a) $\displaystyle\int \frac{x^2+x+3}{x^2+2}\,dx =$

Solution: Polynomial long division gives

$$\frac{x^2+x+3}{x^2+2} = 1 + \frac{x+1}{x^2+2} = 1 + \frac{x}{x^2+2} + \frac{1}{x^2+2}$$

and therefore

$$\begin{aligned}\int \frac{x^2+x+3}{x^2+2}\,dx &= \int 1 + \frac{x}{x^2+2} + \frac{1}{x^2+2}\,dx \\ &= x + \frac{1}{2}\ln|x^2+2| + \frac{1}{\sqrt{2}}\arctan(x/\sqrt{2}) + C.\end{aligned}$$

(b) $\displaystyle\int \sin(2x)e^{5x}\,dx.$

Solution:

$$\begin{aligned}\int \sin(2x)e^{5x}\,dx &= \sin(2x)\frac{e^{5x}}{5} - \int 2\cos(2x)\frac{e^{5x}}{5}\,dx \\ &= \frac{1}{5}\sin(2x)e^{5x} - \frac{2}{5}\left(\cos(2x)\frac{e^{5x}}{5} - \int -2\sin(2x)\frac{e^{5x}}{5}\,dx\right) \\ &= \frac{1}{5}\sin(2x)e^{5x} - \frac{2}{25}\cos(2x)e^{5x} + \frac{4}{25}\int \sin(2x)e^{5x}\,dx \\ \frac{29}{25}\int \sin(2x)e^{5x}\,dx &= \frac{1}{5}\sin(2x)e^{5x} - \frac{2}{25}\cos(2x)e^{5x} + C \\ \int \sin(2x)e^{5x}\,dx &= \frac{5}{29}\sin(2x)e^{5x} - \frac{2}{29}\cos(2x)e^{5x} + C.\end{aligned}$$

(c) $\displaystyle\int_0^{\pi/6} \sec^3(2t)\tan(2t)\,dt =$

Solution: We're going to take $u = \sec(2t)$ so that $du = 2\sec(2t)\tan(2t)\,dt$. We compute $u(0) = 1$ and $u(\pi/6) = \sec(\pi/3) = 2$. Then

$$\begin{aligned}\int_0^{\pi/6} \sec^3(2t)\tan(2t)\,dt &= \int_1^2 \frac{1}{2}u^2\,du \\ &= \frac{u^3}{6}\Big|_1^2 = \frac{8}{6} - \frac{1}{6} = \frac{7}{6}.\end{aligned}$$

S5: Geometric Applications

(a) Compute the area of the surface obtained by taking the curve $x^{2/3} + y^{2/3} = 1$ as x goes from 0 to 1 and rotating it around the y-axis.

Solution: We have $y = (1 - x^{2/3})^{3/2}$, so

$$\begin{aligned} y' &= \frac{3}{2}(1 - x^{2/3})^{1/2} \cdot \frac{-2}{3} x^{-1/3} \\ &= \sqrt{1 - x^{2/3}} x^{-1/3} \\ (y')^2 &= (1 - x^{2/3}) x^{-2/3} = x^{-2/3} - 1 \\ \sqrt{1 + (y')^2} &= \sqrt{x^{-2/3}} = x^{-1/3}. \end{aligned}$$

Then we compute

$$\begin{aligned} L &= \int_0^1 2\pi x \sqrt{1 + (y')^2}\, dx \\ &= 2\pi \int_0^1 x \cdot x^{-1/3}\, dx \\ &= 2\pi \int_0^1 x^{2/3}\, dx = 2\pi \frac{3}{5} x^{5/3} \Big|_0^1 \\ &= \frac{6\pi}{5} \left(1^{5/3} - 0^{5/3}\right) = \frac{6\pi}{5}. \end{aligned}$$

(If you graph the shape, you might notice that there's a symmetry, where we actually have $y = \pm(1 - x^{2/3})^{3/2}$. If you use both halves, you'll get twice this answer.)

(b) Compute the arc length of the curve $(y - 2)^3 = x^2$ between $y = 2$ and $y = 6$ for $x \geq 0$.

Solution: We have $x = (y - 2)^{3/2}$, so $\frac{dx}{dy} = \frac{3}{2}(y - 2)^{1/2}$ and

$$\begin{aligned} L &= \int_2^6 \sqrt{1 + \frac{9}{4}(y - 2)}\, dy \\ &= \frac{8}{27} \left(1 + \frac{9}{4}(y - 2)\right)^{3/2} \Bigg|_2^6 \\ &= \frac{8}{27} \left(10^{3/2} - 1\right). \end{aligned}$$

S6: Differential Equations

(a) Find a general solution to the equation $y' = xye^x$.

Solution:

$$\begin{aligned}\frac{dy}{dx} &= xye^{y}\\ \frac{dy}{y} &= xe^{x}\,dx\\ \ln|y| &= xe^{x} - e^{x} + C\\ y &= e^{xe^{x}-e^{x}}e^{C}.\end{aligned}$$

(b) Find a (specific) solution to the initial value problem $y' = xy - x$ if $y(0) = e + 1$

Solution:

$$\begin{aligned}\frac{dy}{dx} &= xy - x\\ \frac{dy}{y-1} &= x\,dx\\ \ln|y-1| &= x^2/2 + Cy && = e^{x^2/2+C} + 1\end{aligned}$$

Then we have

$$\begin{aligned}e + 1 &= e^{0^2/2+C} + 1\\ 1x^2/2 + CC & && = 1\\ y &= e^{x^2/2+1} + 1.\end{aligned}$$