

Math 1232: Single-Variable Calculus 2
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Recitation 7

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Problem 1. Let $f(x) = x^2$. Let's find the arc length between $x = 0$ and $x = 4$.

- (a) This makes a very reasonable shape. What does the graph look like?
- (b) Set up an integral to compute this arc length. You need to think about the variable of integration, the bounds, and the actual function to integrate.
- (c) What techniques should we use to compute this integral? Where do we get stuck?
- (d) Is there another way we could have set it up?
- (e) Is that integral any easier?

Problem 2. Let $f(x) = \sqrt[3]{3x}$. Take the portion of the graph where $0 \leq y \leq 2$ and rotate it around the y axis.

- (a) Try to sketch a picture of what this will look like.
- (b) Set up an integral to find the surface area. Again, think about the variable of integration, the bounds, and the function. Do you have multiple choices here or just one?
- (c) Can you compute that integral?

Problem 3. Suppose we have a Hooke's Law system of a weight on a spring. Suppose $m = k$, so that we get the differential equation $x''(t) = -x(t)$.

- (a) From class, we know the general solution to this differential equation. What is it?
- (b) Suppose now we start (at time 0) with the weight stationary and displaced by 1 meter. What initial conditions does this correspond to?
- (c) Find the specific solution to this initial value problem.
- (d) What does this describe physically? Does that solution make physical sense?

Problem 4. We can also do a different setup that is not technically an initial value problem. Suppose we have a Hooke's Law setup with a weight on a spring, and $m = k$, so $x(t) = a \sin(t) + b \cos(t)$.

- (a) Suppose the weight starts with a displacement of 2, and at time $t = \pi/4$ the displacement is $\sqrt{8}$. How can we encode that mathematically?
- (b) Why is this not an initial value problem? (We call this a "boundary value problem". Why do you think we call it that?)
- (c) Is this enough information to find a specific solution to the differential equation? What is it?