Math 1232 Spring 2026

Single-Variable Calculus 2

Mastery Quiz 8

Due Thursday, March 20

This week's mastery quiz has two topics. Everyone should submit work on S7. If you have a 2/2 on S6, you don't need to submit it again. (Please check Blackboard to confirm your scores!) This will be the last quiz featuring S6.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it.

Topics on This Quiz

- Secondary Topic 6: Differential Equations
- Secondary Topic 7: Sequences and Series

Name:

Recitation Section:

S6: Differential Equations

(a) Find a general solution to the equation $y' = x^2/y^3$.

Solution:

$$\begin{aligned} \frac{dy}{dx} &= x^2/y^3 \\ y^3\,dy &= x^2\,dx \\ y^4/4 &= x^3/3 + Cy \qquad\qquad = \sqrt[4]{x^3/12 + C/4}. \end{aligned}$$

(b) Find a (specific) solution to the initial value problem $y'/x - y = 1$ if $y(0) = 3$

Solution:

$$\begin{aligned} y'/x &= 1 + y \\ \frac{dy}{1+y} &= x\,dx \\ \ln|1+y| & x^2/2 + C \\ 1 + y &= e^{x^2/2}e^C \\ y &= Ke^{x^2/2} - 1 \\ 3 &= K - 1 \Rightarrow K = 4 \\ y &= 4e^{x^2/2} - 1. \end{aligned}$$

S7: Sequences and Series

(a) Consider the sequence $(a_n) = (6, 2, 2/3, 2/9, \dots)$. Find a formula for the nth term a_n. Compute $\lim_{n\to\infty} a_n$.

Solution: $a_n = \frac{18}{3^n}$ or $a_n = 2 \cdot 3^{2-n}$. We have

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{18}{3^n} = 0.$$

We can justify this by saying the bottom goes to infinity.

(b) Let $b_n = \frac{n!+2}{(n+2)!}$. Compute the first four terms of the sequence, and compute $\lim_{n\to\infty} b_n$, with justification.

Solution:

$$b_1 = \frac{3}{6} = \frac{1}{2} \qquad b_2 = \frac{4}{24} = \frac{1}{6}$$

$$b_3 = \frac{8}{120} = \frac{1}{15} \qquad b_4 = \frac{26}{720} = \frac{13}{360}.$$

To compute the limit, we have

$$\begin{aligned} \lim_{n\to\infty} b_n &= \lim_{n\to\infty} \frac{n! + 2}{(n+2)!} \\ &= \lim_{n\to\infty} \frac{1 + 2/n!}{(n+2)(n+1)} = 0 \\ \text{(or better)} \quad &= \lim_{n\to\infty} \frac{\frac{1}{(n+1)(n+1)} + \frac{2}{(n+2)!}}{1} = \frac{0}{1} = 0. \end{aligned}$$

(c) Compute $\sum_{n=1}^{\infty} \frac{1}{n^2 + 11n + 30}$. Rigorously justify your computation of this limit.

Solution: A partial fractions decomposition gives that $\frac{1}{n^2+11n+30} = \frac{1}{n+5} - \frac{1}{n+6}$. So our series is

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{1}{n+5} - \frac{1}{n+6} &= \lim_{t\to\infty} \sum_{n=1}^{t} \frac{1}{n+5} - \frac{1}{n+6} \\ &= \lim_{t\to\infty} \left((1/6 - 1/5) + (1/7 - 1/6) + \dots \right) \\ &= \lim_{t\to\infty} \frac{1}{6} - \frac{1}{t+6} = \frac{1}{6}. \end{aligned}$$

(d) Compute $\sum_{n=1}^{\infty} \frac{4}{3^{2n}}$.

Solution: This is a geometric series with $a = 4/9$ and $r = 1/9$, so we have

$$\sum_{n=1}^{\infty} \frac{4}{3^{2n}} == \frac{4/9}{1 - 1/9} = 1/2.$$