Math 1232 Spring 2026
Single-Variable Calculus 2
Mastery Quiz 9
Due Thursday, March 26

This week's mastery quiz has two topics. Everyone should submit work on M3. If you have a 2/2 on S7, you don't need to submit it again. (Please check Blackboard to confirm your scores!) This will be the last quiz featuring S7.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it.

Topics on This Quiz

- Major Topic 3: Series Convergence
- Secondary Topic 7: Sequences and Series

Name:

Recitation Section:

M3: Series Convergence

(a) Analyze the convergence of the series $\sum_{n=2}^{\infty} \frac{\ln(n)+n}{n^2-1}$

Solution: You can't really use the limit comparison test here, at least not easily, because the numerator is a bit over-complicated. But you can use the usual comparison test. We know that $n \leq n + \ln(n)$ and $n^2 - 1 < n^2$, so

$$\frac{\ln(n)+n}{n^2-1} \geq \frac{n}{n^2} = \frac{1}{n}.$$

We know that $\sum \frac{1}{n}$ diverges by the p-series test, so our series diverges by the comparison test.

(b) Analyze the convergence of the series $\sum_{n=1}^{\infty} \frac{n^3}{n^4+7}$.

Solution: We have

$$\begin{aligned}\int_1^{\infty} \frac{x^3}{x^4+7}\,dx &= \lim_{t\to\infty} \int_1^t \frac{x^3}{x^4+7}\,dx \\ &= \lim_{t\to\infty} \frac{1}{4}\ln(x^4+7)\Big|_1^t \\ &= \lim_{t\to\infty} \frac{1}{4}\big(\ln(t^4+7) - \ln(8)\big) = \infty.\end{aligned}$$

Thus by the integral test, $\sum_{n=1}^{\infty} \frac{n^3}{n^4+7}$ diverges.

We could also use the limit comparison test here: We see that

$$\lim_{n\to\infty} \frac{\frac{n^3}{n^4+7}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^4}{n^4+7} = 1$$

And sine $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, so does $\sum_{n=1}^{\infty} \frac{n^3}{n^4+7}$.

However, direct comparison won't work—$\frac{n^3}{n^4+7} < \frac{1}{n}$, which doesn't help us.

(c) Analyze the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$.

Solution: We can work this out with the integral test. We have

$$\begin{aligned}\int_1^\infty \frac{1}{x^2+4}\,dx &= \lim_{t\to\infty}\int_1^t \frac{1}{4}\frac{1}{1+(x/2)^2}\,dx\\ &= \lim_{t\to\infty}\frac{1}{2}\arctan(x/2)\Big|_1^t\\ &= \lim_{t\to\infty}\arctan(t/2)-\arctan(1/2) = \pi/2-\arctan(1/2) < \infty.\end{aligned}$$

Since this integral converges, the series must also converge by the integral test.

Alternatively, we can use the (direct) comparison test. We see that $\frac{1}{n^2+4} < \frac{1}{n^2}$. Since $\sum_{n=1}^\infty \frac{1}{n^2}$ converges (by the p-series test), so does $\sum_{n=1}^\infty \frac{1}{n^2+4}$.

Finally, we could use the limit comparison test; it works fine but is a bit clunky. We have

$$\lim_{n\to\infty}\frac{\frac{1}{n^2+4}}{\frac{1}{n^2}} = \lim_{n\to\infty}\frac{n^2}{n^2+4} = 1.$$

Since $\sum_{n=1}^\infty \frac{1}{n^2}$ converges (by the p-series test), so does $\sum_{n=1}^\infty \frac{1}{n^2+4}$.

S7: Sequences and Series

(a) Compute $\displaystyle\sum_{n=1}^\infty \frac{-1}{n^2+5n+6}$. Rigorously justify your computation of this limit.

Solution: This is the same as

$$\begin{aligned}\sum_{n=1}^\infty \frac{1}{n+3}-\frac{1}{n+2} &= \lim_{t\to\infty}\sum_{n=1}^t \frac{1}{n+3}-\frac{1}{n+2}\\ &= \lim_{t\to\infty}\left((1/4-1/3)+(1/5-1/4)+\dots\right)\\ &= \lim_{t\to\infty}\frac{1}{t+3}-\frac{1}{3} = \frac{-1}{3}.\end{aligned}$$

Thus this sum diverges to infinity.

(b) Consider the sequence $(a_n) = (3, 6/2, 9/6, 12/24, 15/120, \dots)$. Find a formula for the nth term a_n. Compute $\lim_{n\to\infty} a_n$.

Solution: We have $a_n = \frac{3n}{n!}$, and thus

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{3n}{n!} = \lim_{n\to\infty}\frac{1}{(n-1)!} = 0.$$

(c) Let $b_n = \tan\left(\frac{(2n-1)\pi}{4}\right)$. Compute the first four terms of the sequence, and compute $\lim_{n\to\infty} b_n$, with justification.

Solution:

$$b_1 = \tan(\pi/4) = 1 \qquad b_2 = \tan(3\pi/4) = -1$$
$$b_3 = \tan(5\pi/4) = 1 \qquad b_4 = \tan(7\pi/4) = -1.$$

We see that this sequence just alternates between 1 and -1, so it will never converge. The limit does not exist.

(d) Compute $\displaystyle\sum_{n=1}^{\infty} \frac{(-4)^{n-1}}{5^n}$.

Solution: This is a geometric series with $a = \frac{1}{5}$ and $r = \frac{-4}{5}$. Since $|r| = 4/5 < 1$ this series converges, to $\frac{1/5}{1+4/5} = \frac{1}{9}$.