Math 2233:Multivariable Calculus

George Washington University Summer 2026

Recitation 12

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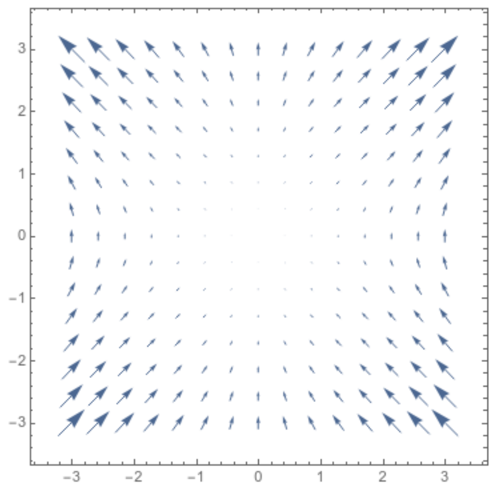
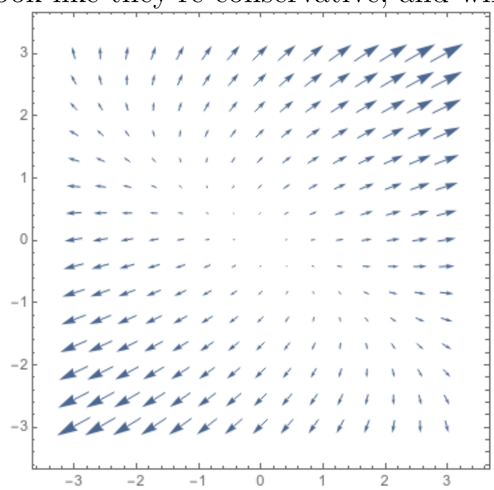
Problem 1. Which of these vector fields look like they're conservative, and why?





Problem 2. (a) Consider the vector fields $\vec{F}(x,y) = (x+y)\vec{i}+x\vec{j}$ and $\vec{G}(x,y) = (x+y)\vec{i}+y\vec{j}$. One of these is conservative. Without computing anything, can you tell which is conservative, and why?

(b) Try to sketch out plots of $\vec{F}$ and $\vec{G}$. Now can you guess which is conservative?

(c) Set up the differential equations to find a potential field for $\vec{F}$. Either produce the potential, or prove it doesn't exist.

(d) Do the same for $\vec{G}$.

Problem 3. Let $f(x,y,z) = xy + zx^2$. Compute $\int_C \nabla f \, ds$ where C is parametrized by the curve $\vec{r}(t) = (2t-1, t^4+t, \sin(\pi t))$ for $t \in [0,2]$.

Problem 4. Let's compute the circulation density of $\vec{G}(x,y) = (x+y)\vec{i}+y\vec{j}$ at the origin.

(a) Can we parametrize a small circle of radius a centered at the origin?

(b) Compute the line integral of G around this circle.

(c) Use that to compute the circulation *density*.

We can define an operator called the *curl*. On Monday we'll talk about why we might care about it, but for right now we can give a formula:

Definition 0.1. Let $\vec{F}(x,y,z) = F_1(x,y,z)\vec{i} + F_2(x,y,z)\vec{j} + F_3(x,y,z)\vec{k}$ be a vector field in $\mathbb{R}^2$ or $\mathbb{R}^3$. We define the *curl* of $\vec{F}$ to be

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z} \right) \vec{i} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x} \right) \vec{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \vec{k}.$$

If $\vec{F}$ is a vector field from $\mathbb{R}^2 \to \mathbb{R}^2$, this reduces to

$$\nabla \times F = \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \vec{k}$$

and we often treat it as a scalar quantity.

(a) Compute $\nabla \times (x\cos(y))\vec{i} + xy^2\vec{j}$.

(b) Compute $\nabla \times (xy)\vec{i} + (yz)\vec{j} + (xz)\vec{k}$.

(c) Compute $\nabla \times (x+y)\vec{i} + x\vec{j}$

(d) Compute $\nabla times (x+y)\vec{i} + y\vec{j}$.