

Math 2233 Summer 2026
Multivariable Calculus
Mastery Quiz 3 Worksheet
Due Wednesday, July 8

This week's mastery quiz has four topics. (This is a lot; we should never have this many at once again.) Everyone should prepare M2 and S3. You may or may not still need to prepare M1 and S2, but I encourage everyone to work through them to compare to the solutions that will be posted later.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, you should justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

On Wednesday, we will end class with a short quiz, containing questions drawn from this worksheet. You may not use or consult any materials or notes during Wednesday's quiz; make sure you work through these problems carefully, and understand them well enough that you can replicate that work in class.

Topics on This Quiz

- Major Topic 1: Vector Functions
- Major Topic 2: Partial Derivatives
- Secondary Topic 2: Vector Functions
- Secondary Topic 3: Multivariable Functions

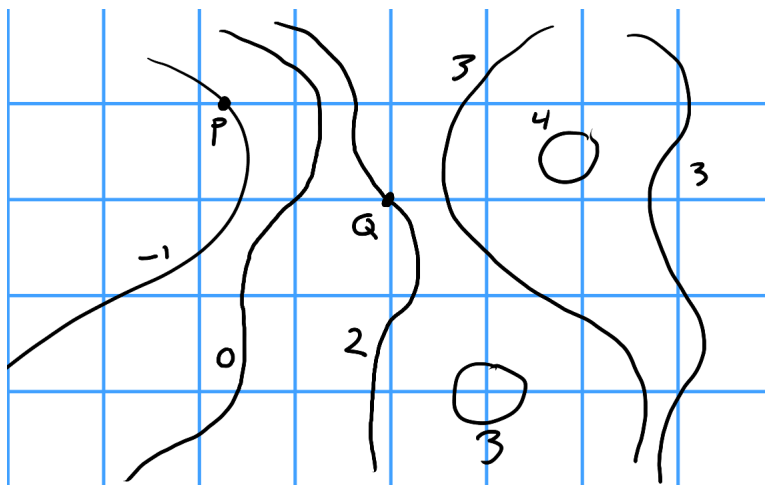
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M1: Vectors

- (a) Find the area of the parallelogram with vertices $(0, 0, 0)$, $(3, 1, 2)$, $(2, 4, 3)$, $(5, 5, 5)$.
- (b) Find the orthogonal decomposition of $\vec{v} = 4\vec{i} + \vec{j} - \vec{k}$ with respect to $\vec{u} = 2\vec{i} - \vec{j} + 3\vec{k}$.
- (c) Find $\cos \theta$ where θ is the angle between $\vec{u} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{v} = \vec{i} - \vec{j} + \vec{k}$.

M2: Partial Derivatives

- (a) Below is a contour plot of the function $f(x, y)$.
- Sketch the gradient vector at P .
 - Estimate $\frac{\partial f}{\partial x}$ at the point Q . Explain your reasoning in a sentence or so.



- (b) Let $h(x, y, z) = e^{xyz^2} - \frac{x+yz}{y^2-xz}$. Compute $\nabla h(x, y, z)$.
- (c) Give an equation for the plane tangent to $f(x, y) = 3 + x^2y + \ln(x + xy)$ at the point $(1, 0)$.

S2: Vector Functions

- (a) Suppose two particles follow the paths $\vec{r}_1(t) = (3t, t^2, 4 - t)$ and $\vec{r}_2(t) = (t^2 + 2, 6 - t, 3t - 4)$. Do the two particles collide? Do their paths intersect?
- (b) (i) Find a parametric equation for a particle moving in a straight line, starting at $(5, 2, -2)$ and moving towards $(4, -3, 2)$.

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- (ii) Suppose another particle follows the path $\vec{r}_2(t) = (t - 3, 5 - t, t)$. Does this particle's path intersect the path of the particle from part (a)?
- (c) Find an equation for the line tangent to the curve $\vec{r}(t) = (3t, \ln(t^2 + 1), 5t^2 + 2)$ at the time $t = 3$.

S3: Multivariable Functions

- (a) Let $g(x, y) = 1 + e^{x^2 + y^2}$. Sketch a clearly labeled contour diagram that includes contours for $c = 1, c = 3, c = 5$. In a few words can you describe the shape of the graph of g ?
- (b) Let $h(x, y, z) = \frac{x^2 - y^2}{xz - yz}$. Compute $\lim_{(x, y, z) \rightarrow (1, 1, 1)} h(x, y, z) =$
- (c) Let $f(x, y) = 3x^2 + xy - y$. Sketch cross-sections of f for $x = -1, 0, 1$ and $y = -2, 0, 2$.