Math 2233:Multivariable Calculus
George Washington University Summer 2026
Recitation 4

Jay Daigle

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Problem 1. Here's a limit that's a little interesting. Let $f(x,y) = \dfrac{\sin(x^2+y^2)}{x^2+y^2}$.

(a) What is $f(1,1)$? What is $f(3,-2)$?

(b) What is $\lim_{(x,y)\to(0,0)} f(x,y)$? Why?

Problem 2. Let $g(x,y) = \frac{x^2y}{x^4+y^2}$. We want to think about $\lim_{(x,y)\to(0,0)} g(x,y)$.

(a) Can we make a useful heuristic argument here? How many zeroes are in the numerator? In the denominator?

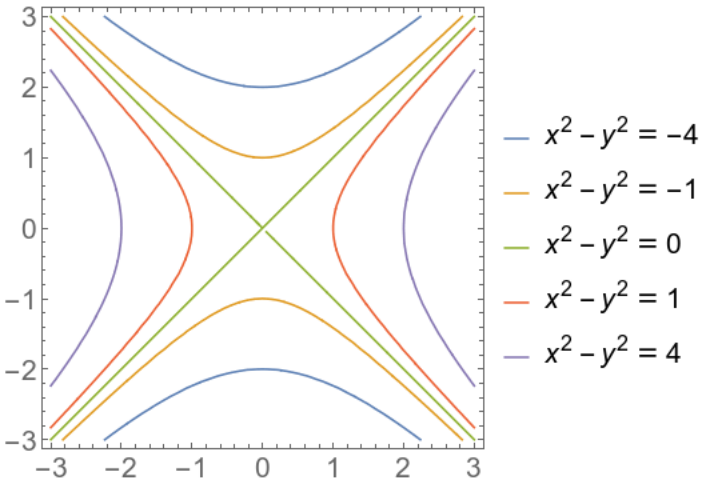
(b) Suppose we approach along the path $y=0$, as we did in class. Then what limit do we get? I.e. what is the limit $\lim_{x\to 0} g(x,0)$?

(c) What if we approach along the path $x=0$?

(d) More generally, let's approach along the line $y=mx$, for some real number $m \neq 0$. Then what is the limit? Do you have a guess of what this limit is?

(e) Now let's take $y=x^2$, so we have $\lim_{x\to 0} g(x,x^2)$. What limit do we get now? What does this tell you?

Problem 3. Consider the following contour diagram:



(a) Look at the point $(0, 2)$. Is $f_x(0, 2)$ positive, negative, or zero? Is $f_y(0, 2)$ positive, negative, or zero? Why?

(b) Slightly trickier: Is $f_x(1, 1)$ positive, negative, or zero? What about $f_y(1, 1)$?

(c) Let $\vec{u} = \vec{i} + \vec{j}$ and $\vec{v} = \vec{i} - \vec{j}$. Is $f_{\vec{u}}(1, 0)$ positive, negative, or zero? What about $f_{\vec{v}}(1, 0)$?

(d) Is $f_{\vec{u}}(-1, -1)$ positive, negative, or zero? What about $f_{\vec{v}}(-1, -1)$?

(e) Where do you think $f_x(x, y) = 0$?

(f) Where do you think $f_y(x, y) = 0$?

(g) Where is $f_{\vec{u}}(x, y) = 0$?

(h) Is there any point where you might worry the partial derivatives are undefined?

(i) This function is given by the formula $f(x, y) = x^2 - y^2$. Compute the partial derivatives. Does this confirm what you found in the previous parts?

Problem 4. Let $g(x, y) = \sin(x) + \sin(2y) + \sin(x + y)$.

(a) Find a linear approximation to g at the point $(0, 0)$.

(b) Find an equation for the tangent plane at the point $(0, 0, 0)$.

(c) Estimate $g(.1, .2)$.