Math 2233 Summer 2026
Multivariable Calculus
Mastery Quiz 4 Worksheet
Due Monday, July 13

This week's mastery quiz has three topics. Pretty much everyone should prepare M2. You may or may not still need to prepare M1 and S3, but I encourage everyone to work through them to compare to the solutions that will be posted later. This will be the last quiz featuring M1 or S3.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, you should justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

On Wednesday, we will end class with a short quiz, containing questions drawn from this worksheet. You may not use or consult any materials or notes during Wednesday's quiz; make sure you work through these problems carefully, and understand them well enough that you can replicate that work in class.

Topics on This Quiz

- Major Topic 1: Vector Functions
- Major Topic 2: Partial Derivatives
- Secondary Topic 3: Multivariable Functions

Name:

Name:

M1: Vectors

(a) Let $\vec{v} = 2\vec{i} + 3\vec{j} + 2\vec{k}$ and $\vec{u} = \vec{i} - \vec{j} + \vec{k}$. Compute the orthogonal decomposition of $\vec{v}$ with respect to $\vec{u}$. That is, write $\vec{v} = \vec{v}_{\text{parallel}} + \vec{v}_{\perp}$.

(b) Find the area of the **triangle** with vertices $(0,0,0), (5,3,1), (7,2,2)$.

(c) Find $\cos\theta$ where θ is the angle between $\vec{u} = -2\vec{i} + 4\vec{j} + \vec{k}$ and $\vec{v} = 3\vec{i} + \vec{j} + 4\vec{k}$.

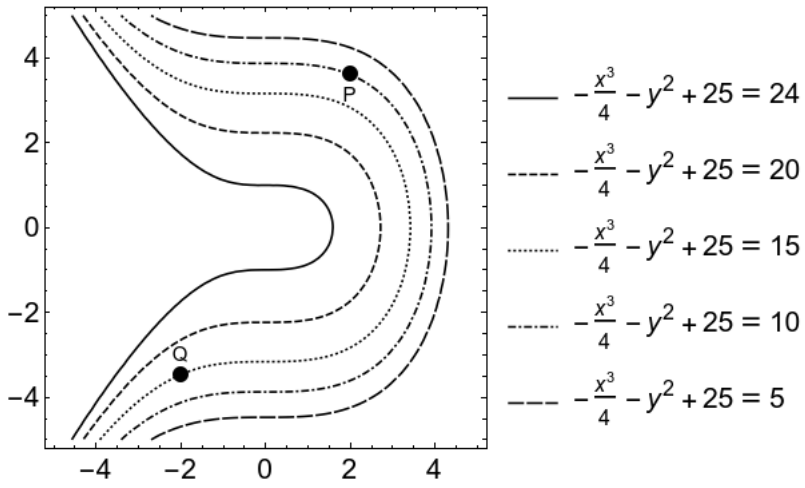
M2: Partial Derivatives

(a) Let $f(x,y,z) = \ln(xy + z)$. Find the directional derivative in the direction $-\vec{i} - \vec{j} + \vec{k}$ at the point $(0,3,1)$.

(b) Find all second partial derivatives of $g(x,y) = \frac{x^2-3}{y+1}$.

(c) Below is a contour plot of the function $h(x,y)$.

- Sketch the gradient vector at P.
- Sketch the gradient vector at Q.
- Label a point on the diagram where $\frac{\partial h}{\partial y} = 0$.



S3: Multivariable Functions

(a) Let $f(x,y) = x^2\sin(y) + 1$. Sketch cross-sections of f for $x = 0, 1, 2$ and $y = -\pi/2, 0, \pi/2$.

(b) Let $g(x, y) = 4x^2 + y^2 - 4$. Sketch a clearly labeled contour diagram that includes contours for $c = -3, c = 0, c = 5$. In a few words can you describe the shape of the graph of g?

(c) Let $h(x, y) = \frac{x^2y^2}{2x^4+y^4}$. Show that $\lim_{(x,y)\to(0,0)}$ does not exist.