

Math 2233 Summer 2026
Multivariable Calculus
Mastery Quiz 6 Worksheet
Due Monday, July 20

This week's mastery quiz has three topics. This is the first quiz featuring M4, and the last featuring M2.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, you should justify your answers and show your work. Do not just write “yes” or “no” or give a single number.

On Wednesday, we will end class with a short quiz, containing questions drawn from this worksheet. You may not use or consult any materials or notes during Wednesday's quiz; make sure you work through these problems carefully, and understand them well enough that you can replicate that work in class.

Topics on This Quiz

- Major Topic 2: Partial Derivatives
- Major Topic 3: Optimization
- Major Topic 4: Integration

Name:

M2: Partial Derivatives

- (a) Give an equation for the plane tangent to $f(x, y) = \cos(x^3y^2) - \frac{x}{y}$ at the point $(0, 3)$.
- (b) Find all three second partial derivatives of $g(x, y) = x^2y + xy^3$.
- (c) Let $f(x, y, z) = \frac{e^{x\sqrt{y}}}{z}$. Find the directional derivative in the direction $\vec{i} + 3\vec{j} - 2\vec{k}$ at the point $(0, 1, 1)$.

M3: Optimization

- (a) Find the maximum and minimum values of $g(x, y, z) = y^2 - 10z$ subject to the constraint $x^2 + y^2 + z^2 = 36$.
- (b) Find and classify the critical points of $f(x, y) = x^2 + xy^2 + y^2$.
- (c) Find (but don't classify) the critical points of $g(x, y, z) = x^3 + y^3 - 3x^2 - y^2 - z^2 + 2z - 1$.

M4: Integration

- (a) Find the volume of the solid bounded above by $f(x, y) = e^{-x^2}$, below by the plane $z = 0$, and over the triangle formed by $x = 1, y = 0, y = x$.
- (b) Sketch the region of integration and compute $\int_{-1}^0 \int_{-\sqrt{y+1}}^{\sqrt{y+1}} y^2 dx dy$. (Do not use a calculator!)
- (c) Integrate the function $f(x, y, z) = xyz$ over the region bounded by $x = 0, x = 4, y = 0, z = 0$, and $y = 4 - z^2$.