

Math 2233 Summer 2026
Multivariable Calculus
Mastery Quiz 8 Worksheet
Due Monday, July 27

This week's mastery quiz has three topics. This is the first quiz featuring S4, and the last quiz featuring M3.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, you should justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

On Wednesday, we will end class with a short quiz, containing questions drawn from this worksheet. You may not use or consult any materials or notes during Wednesday's quiz; make sure you work through these problems carefully, and understand them well enough that you can replicate that work in class.

Topics on This Quiz

- Major Topic 3: Optimization
- Major Topic 4: Integration
- Secondary Topic 4: Integral Applications

Name:

M3: Optimization

- (a) Find and classify the critical points of $f(x, y) = (3x + 4x^3)(y^2 + 2y)$.
- (b) Find the maximum and minimum values of $f(x, y) = xy$ subject to the constraint that $x^2 + 4y^2 \leq 1$.

M4: Integration

- (a) Sketch the region of integration and compute $\iint_R y\sqrt{x^2 + y^2} dA$ where R is the region given by $x^2 + y^2 \leq 4$ and $0 \leq y \leq x$.
- (b) Let R be the parallelogram with vertices $(0, 0), (1, 2), (3, 3), (4, 5)$. Find a transformation that translates to the square with vertices $(0, 0), (0, 1), (1, 0), (1, 1)$. Use this transformation to compute $\iint_R xy dA$.
- (c) Find the volume of the solid bounded by the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 6$ (above the plane $z=0$).

S4: Integral Applications

- (a) Find the mass of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$, and $x + 2y + 3z = 6$ if the density is given by $\delta(x, y, z) = z$.
- (b) Suppose an insurer has found that if x is the age of a car involved in an accident, and y is the length of time since the car was insured, then these variables have a joint probability density function of

$$p(x, y) = \begin{cases} c(5 - xy) & 1 \leq x \leq 5 \quad \text{and} \quad 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Find the value of c that makes this a probability density function.
- (ii) What is the expected age of a car involved in an accident?
- (iii) What is the average duration of the insurance policy at the time of the accident?